Deep learning for early warning signals of bifurcations

Dynamics Days Europe
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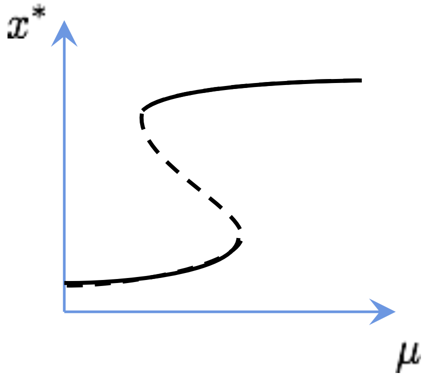
Big data and bifurcations

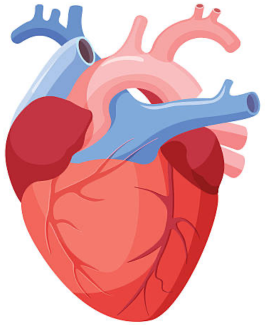
1.)

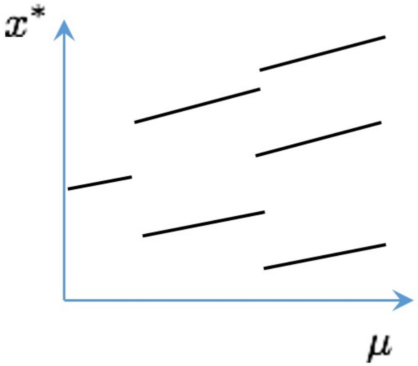


1 measurement / minute
1 year = **O(10^7)** data points

North Temperate Lakes
US Long-Term Ecological Research Network

2.)

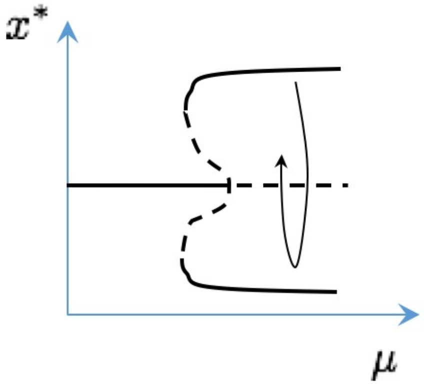


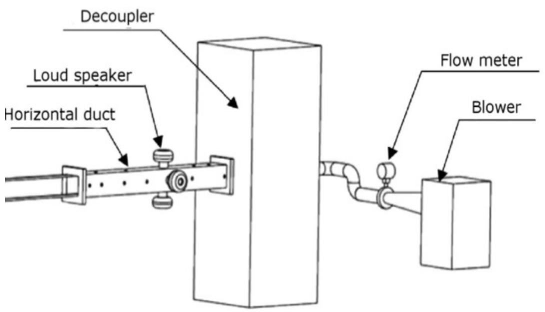


Sampling rate 250Hz
2 weeks = **O(10^8)** data points

icentia™

3.)





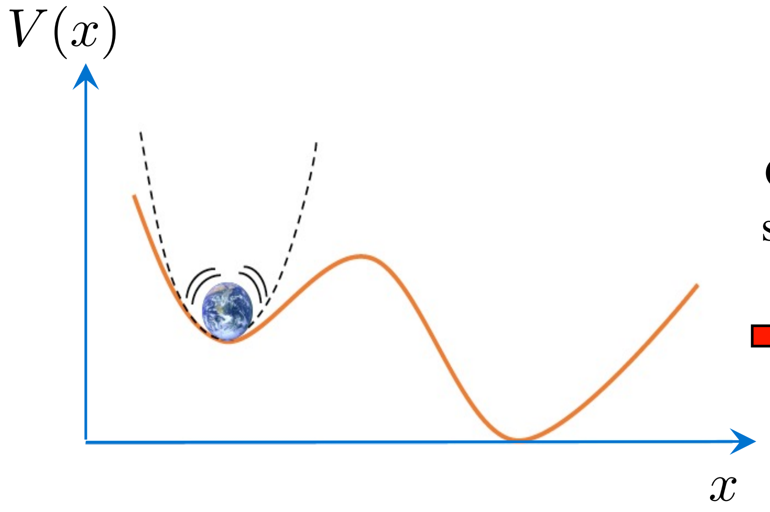
Sampling rate 10kHz
20 minutes = **O(10^7)** data points

R. I Sujith, IIT Madras

Early warning signals based on critical slowing down

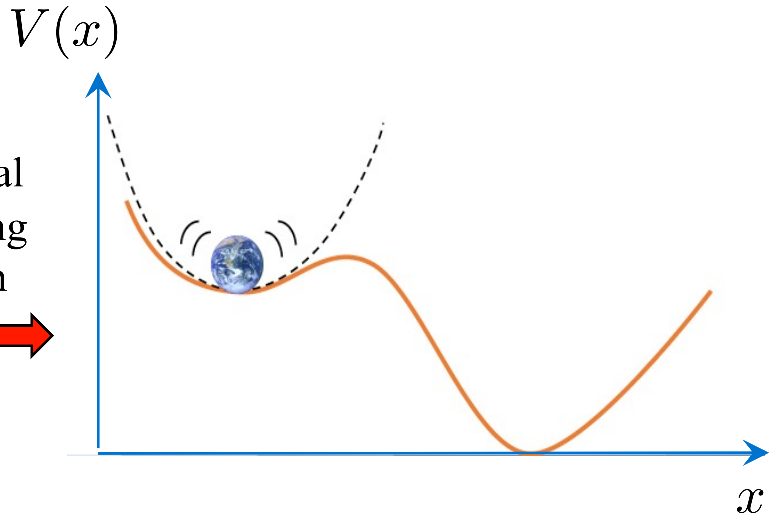
Scheffer *et al.* Early-warning signals for critical transitions. *Nature* (2009)

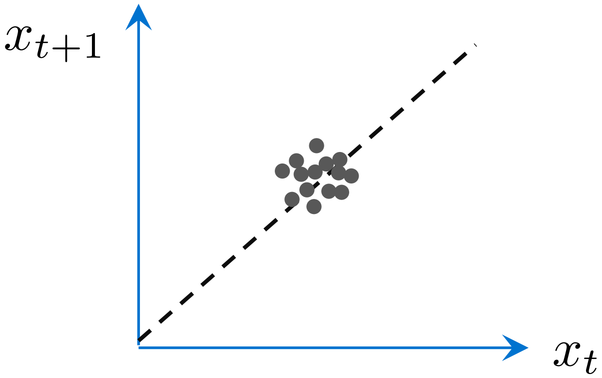
Far from bifurcation

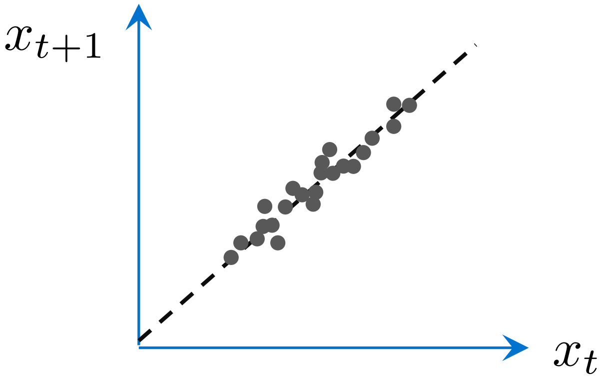


Critical slowing down

Close to bifurcation







Dominant eigenvalue → 0

Increased return time following perturbation

Local flattening of potential landscape

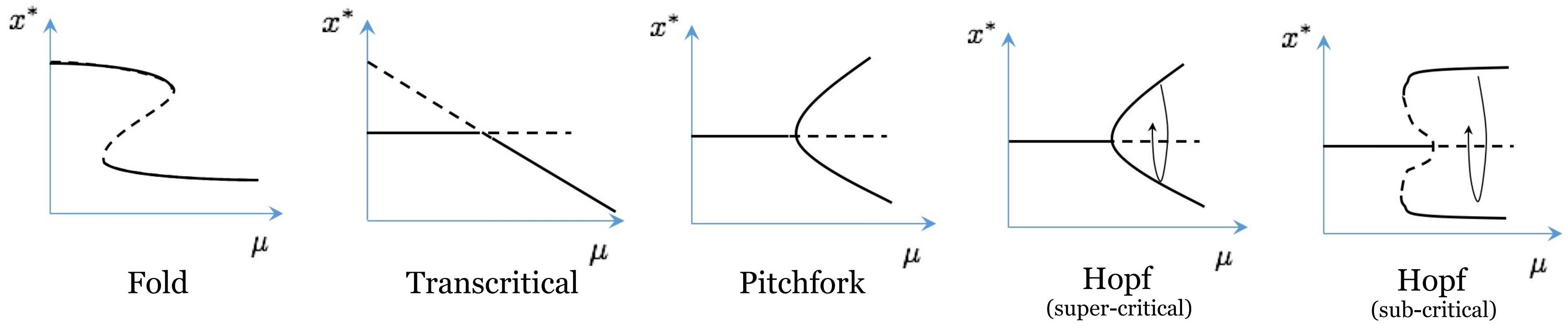
Stochasticity

Increased variance

Increased autocorrelation

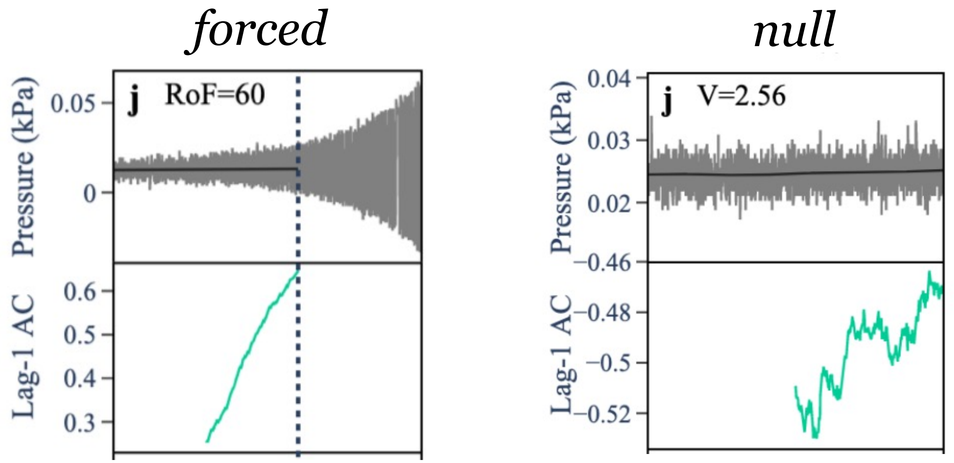
Two limitations of CSD-based EWS

1. Not specific to different bifurcations[1]



2. A qualitative measure[2]

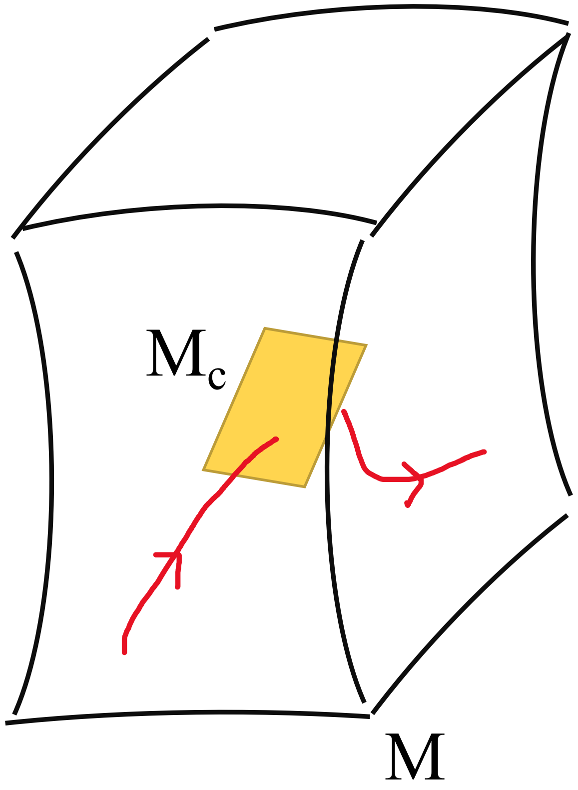
Conventional EWS are based on qualitative changes in metrics, not absolute values, making it difficult to attribute statistical significance to detection.



[1]**Kefi et al.,** Early warning signals also precede non-catastrophic transitions. Oikos (2013)

[2]**Boettiger et al.,** Quantifying limits to detection of early warning signals for critical transitions. R. Soc. Interface (2012)

Why deep learning for bifurcation prediction?



M: high-dimensional manifold containing all possible states

M_C: low-dimensional **centre manifold**

Deep learning is state-of-the-art for **pattern recognition**[1]

Is bifurcation prediction pattern recognition?

Hypothesis*:*
Deep learning can detect 'patterns' associated with a given bifurcation from time series data of a system approaching this bifurcation

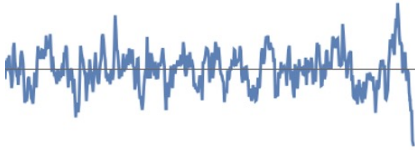
…but will require a **lot** of data

[1]**Lecun, Bengio and Hinton,** Deep learning. Nature (2015)

How deep learning for bifurcation prediction?

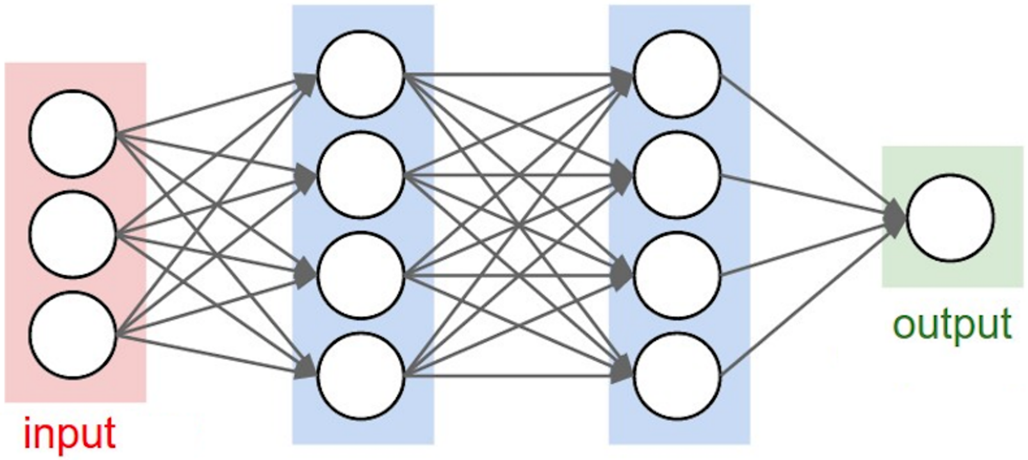
Our approach: Turn it into a classification problem.

$$\vec{y} = f(\vec{x})$$

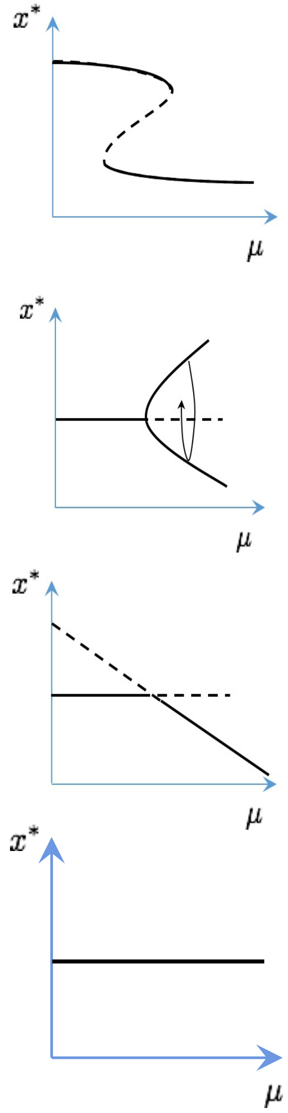


$\vec{x}$ $\qquad f \qquad$ $\vec{y}$

$$\vec{x} = \begin{pmatrix} x_{t_1} \\ x_{t_2} \\ \vdots \\ x_{t_n} \end{pmatrix}$$



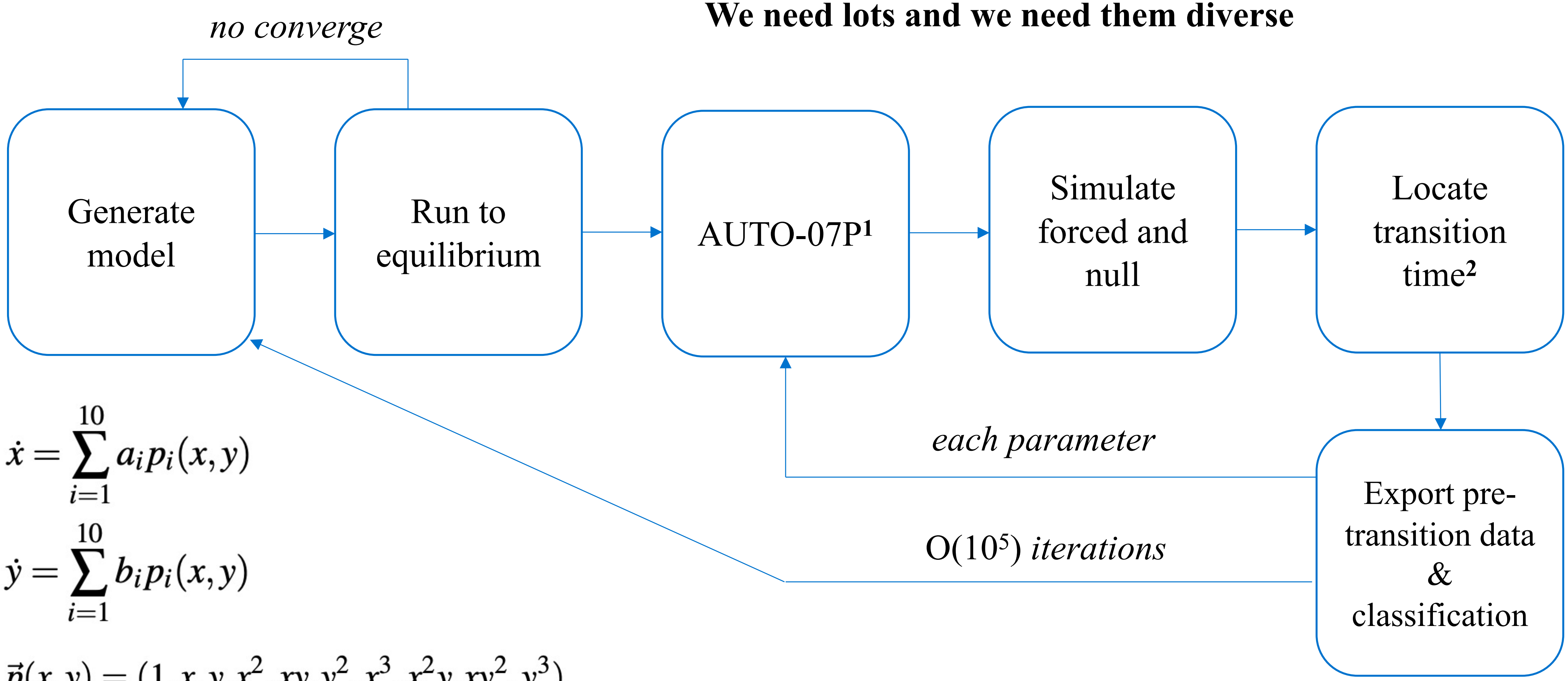
$$\vec{y} = \begin{pmatrix} P(\text{Fold}) \\ P(\text{Hopf}) \\ P(\text{Transcritical}) \\ P(\text{Null}) \end{pmatrix}$$



f: CNN-LSTM – Convolutional Neural Network—Long short-term memory

Generating the training data

We need lots and we need them diverse



$$\dot{x} = \sum_{i=1}^{10} a_i p_i(x,y)$$

$$\dot{y} = \sum_{i=1}^{10} b_i p_i(x,y)$$

$$\vec{p}(x,y) = (1,x,y,x^2,xy,y^2,x^3,x^2y,xy^2,y^3).$$

$$a_i, b_i \sim \mathcal{N}(0,1).$$

[1] **Doedel et al.,** AUTO-07p: Continuation and bifurcation software for ordinary differential equations (2007)

[2] **Truong et al.,** Selective review of offline change point detection methods. Signal Processing (2020)

Training the neural network(s)

500,000 simulations
(125,000 of each type)

95% training

4% validation

1% test

Neural network architecture

Conv1D (50 filters, 12 kernel size)
↓
Dropout (10%)
↓
MaxPooling1D (pool size 2)
↓
LSTM (50 mem cells)
↓
Dropout (10%)
↓
LSTM (10 mem cells)
↓
Dropout (10%)
↓
Dense (4, softmax)

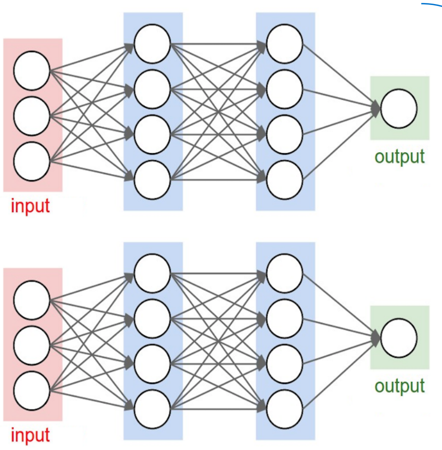
Processing

- Simulations are detrended
- Residuals are normalised and padded with zeros

Nitty gritty

- Learning rate = 5×10^{-4}
- Trained for 15,000 epochs
- Hyperparameters chosen through grid sweeps
- Adam optimization algorithm

Two variants:



ensemble average

	F1 score	Precision	Recall
500-classifier	84.2%	84.4%	84.2%
1500-classifier	88.2%	88.3%	88.3%

K + TensorFlow

EWS in ecological models

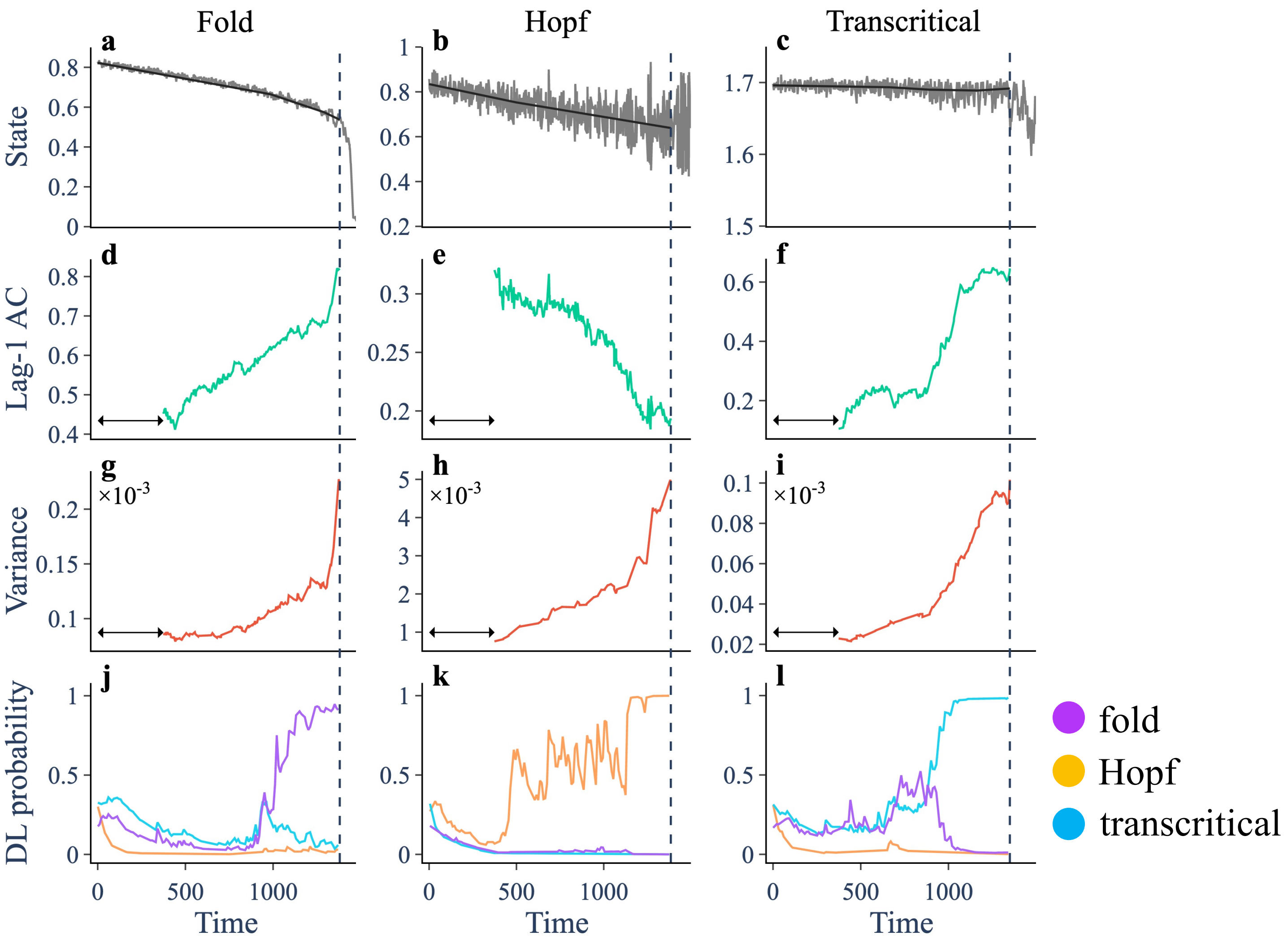
1. May's harvesting model[1]

$$\frac{dx}{dt} = rx\left(1-\frac{x}{k}\right) - h\frac{x^2}{s^2+x^2} + \sigma\xi(t)$$

2. Rozenzweig-MacArthur consumer-resource model[2]

$$\frac{dx}{dt} = rx\left(1-\frac{x}{k}\right) - \frac{axy}{1+ahx} + \sigma_1\xi_1(t)$$
$$\frac{dy}{dt} = \frac{eaxy}{1+ahx} - my + \sigma_2\xi_2(t)$$

- Decreasing lag-1 AC prior to Hopf when data spacing on similar order to period of oscillations[3]

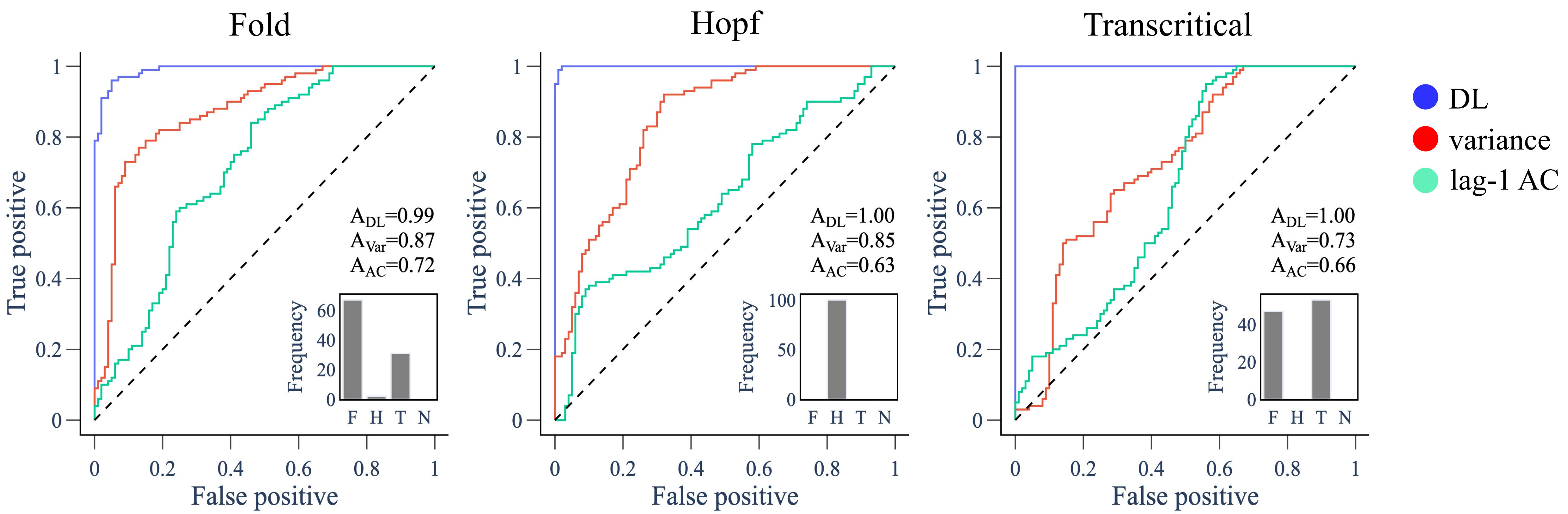


[1]**May, R. M.** Thresholds and breakpoints in ecosystems with a multiplicity of stable states. *Nature* (1977)

[2]**Rosenzweig, M. L. & MacArthur, R. H.** Graphical representation and stability conditions of predator-prey interactions. *The Am. Nat* (1963)

[3]**Bury, T. M *et al.*** Detecting and distinguishing tipping points using spectral early warning signals. *J. Royal Soc. Interface* (2020)

Prediction performance for ecological models



Discrimination metric

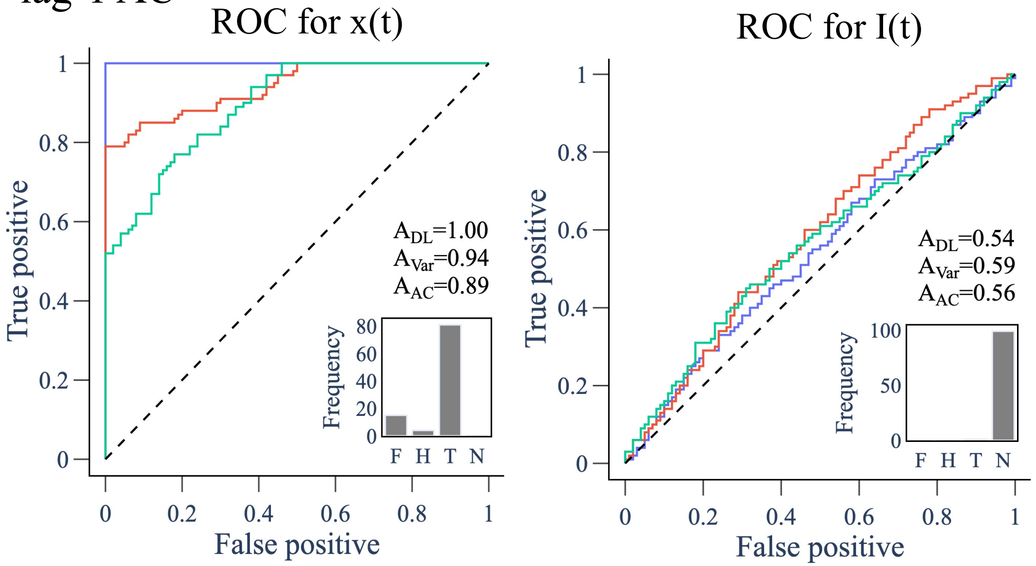
EWS: Kendall tau value

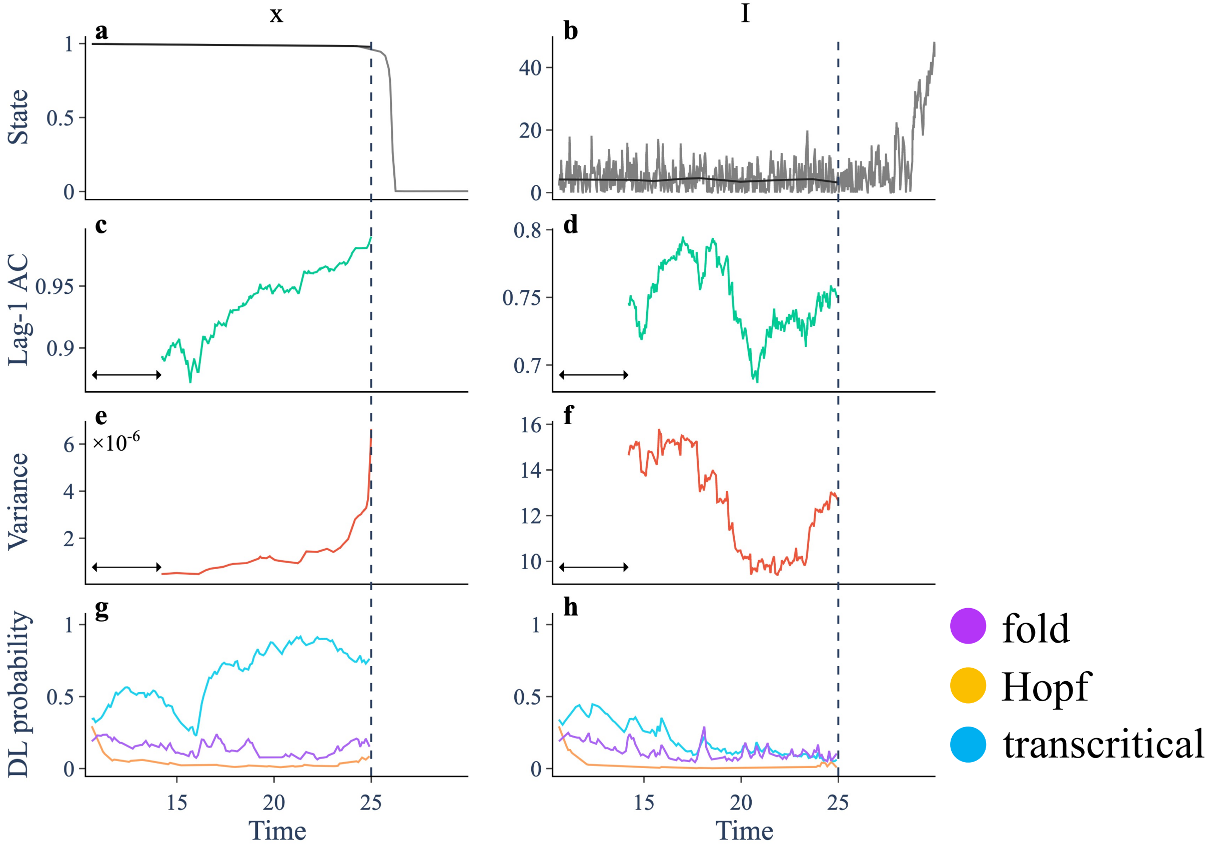
DL: Sum of DL probabilities for bifurcations

EWS in the SEIRx model[1]

$$\frac{dS}{dt} = \mu N(1-x) - \mu S - \beta SI/N + \sigma_1 \xi_1(t),$$
$$\frac{dE}{dt} = \beta SI/N - (\sigma+\mu)E + \sigma_2 \xi_2(t),$$
$$\frac{dI}{dt} = \sigma E - (\gamma+\mu)I + \sigma_3 \xi_3(t),$$
$$\frac{dR}{dt} = \mu x + \gamma I - \mu R + \sigma_4 \xi_4(t),$$
$$\frac{dx}{dt} = \kappa x(1-x)(-\omega + I + \delta(2x-1)) + \sigma_5 \xi_5(t),$$

DL
variance
lag-1 AC





EWS in I(t) are little better than random.

Why? Dominant eigenvector (direction of CSD) lies perpendicular to the measured variable in state-space.[2]

[1]**Pananos, A. D. *et al.*** Critical dynamics in population vaccinating behavior. *PNAS* (2017)

[2]**Boerlijst *et al.*,** Catastrophic collapse can occur without early warning: examples of silent catastrophes in structured ecological models. *PLoS one* (2013)

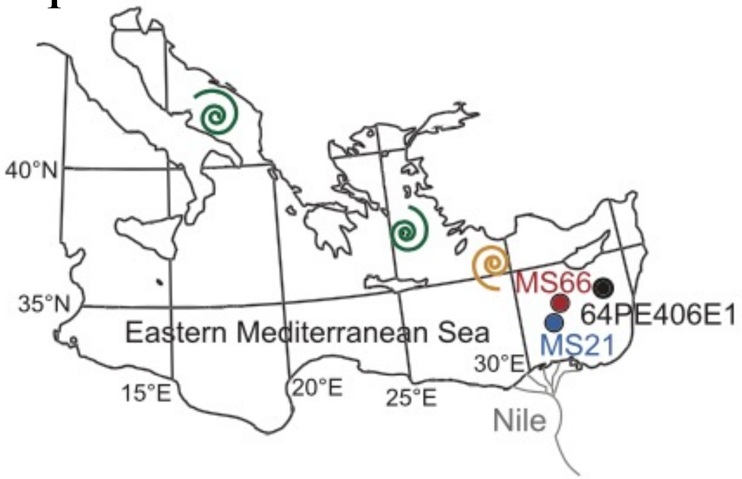
EWS in empirical systems

Anoxic transitions[1] (Fold)
- 26 forced trajectories
- 26 null trajectories (generated AR1)

Thermoacoustic instability (Hopf):
- 19 forced trajectories
- 10 null trajectories (experiment data)

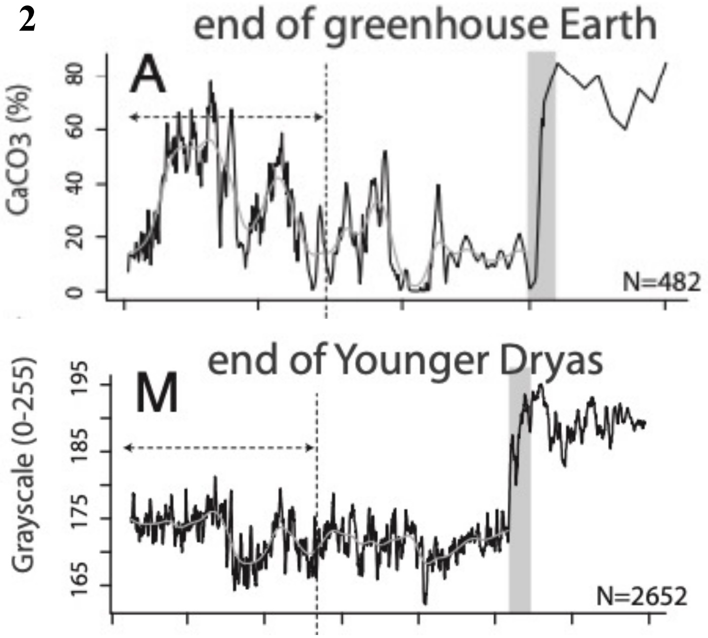
Paleoclimate transitions[2] (?)
- 7 forced trajectories
- 70 null trajectories (generated AR1)

1



(Image courtesy of R. Sujith)

2



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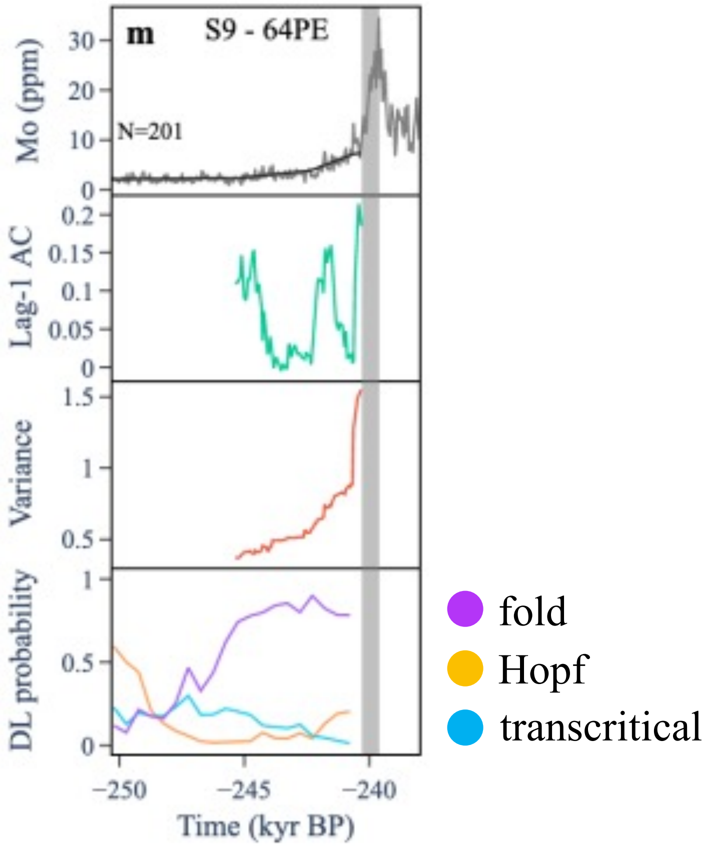

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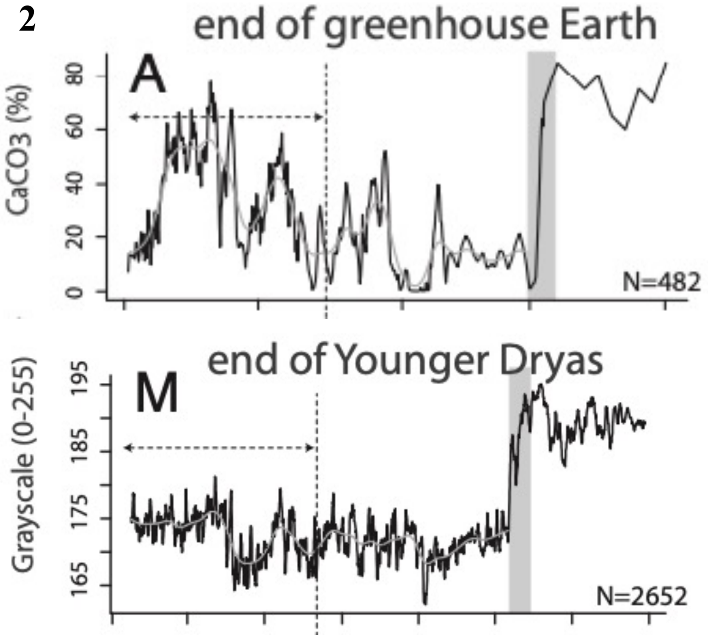

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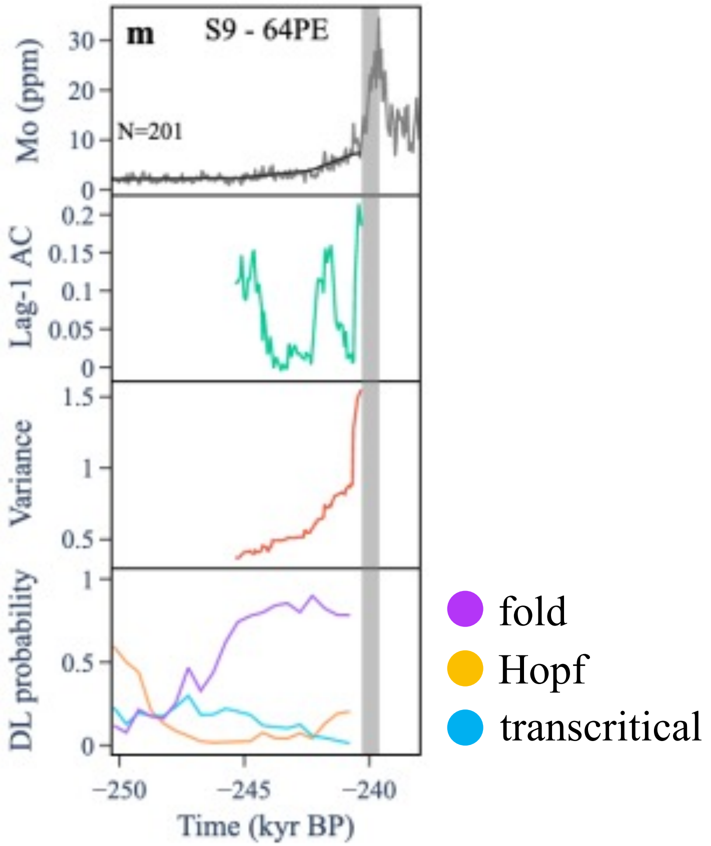
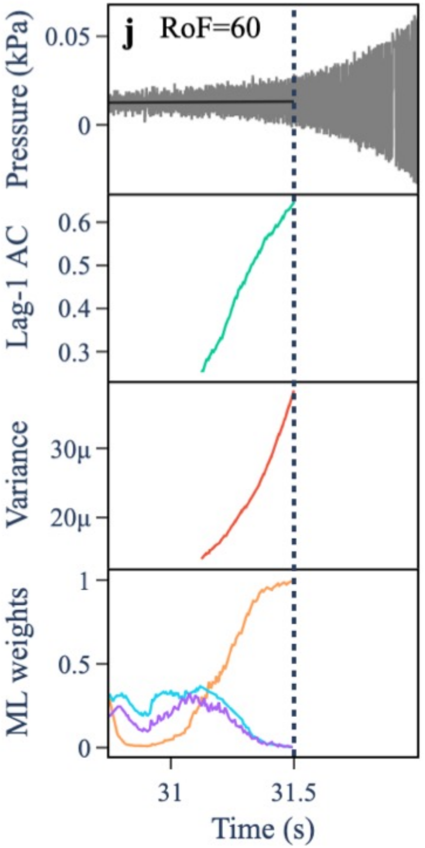
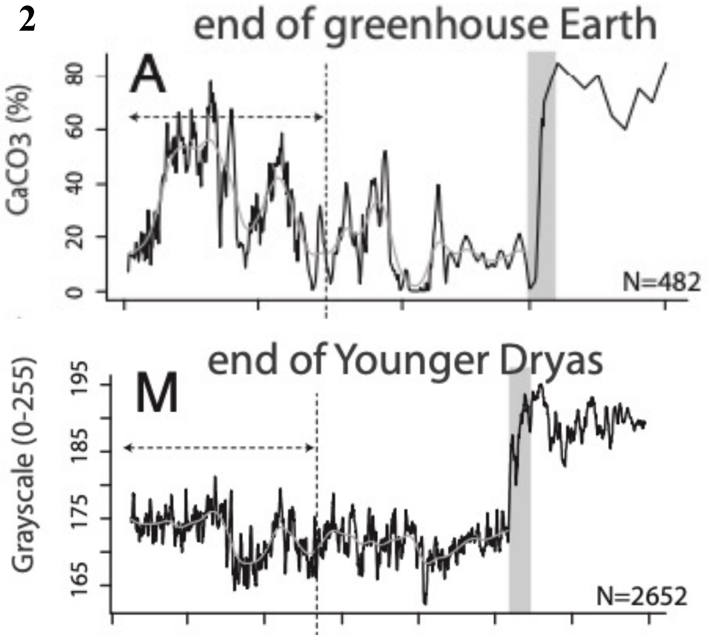




2



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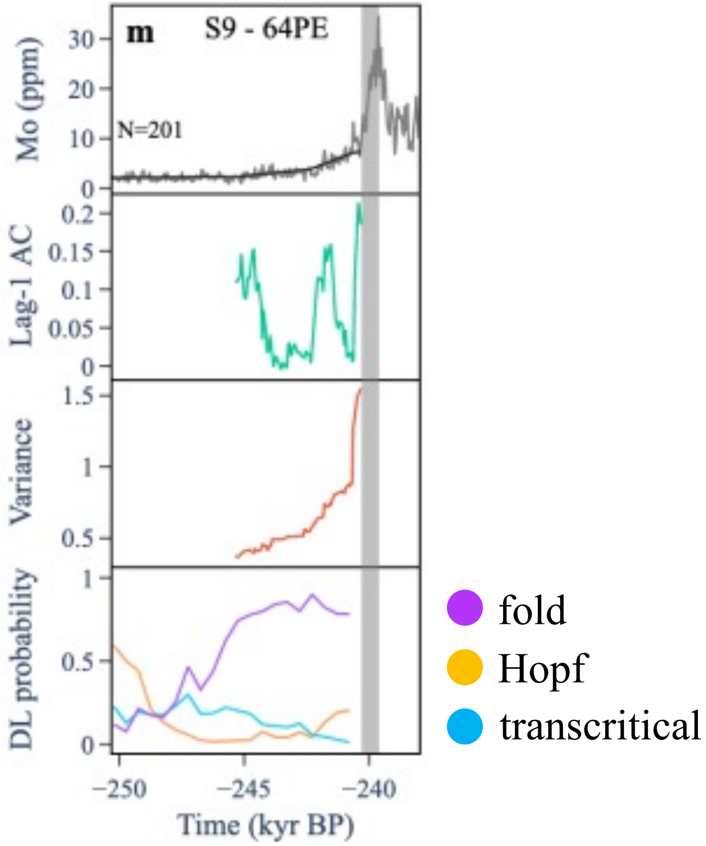

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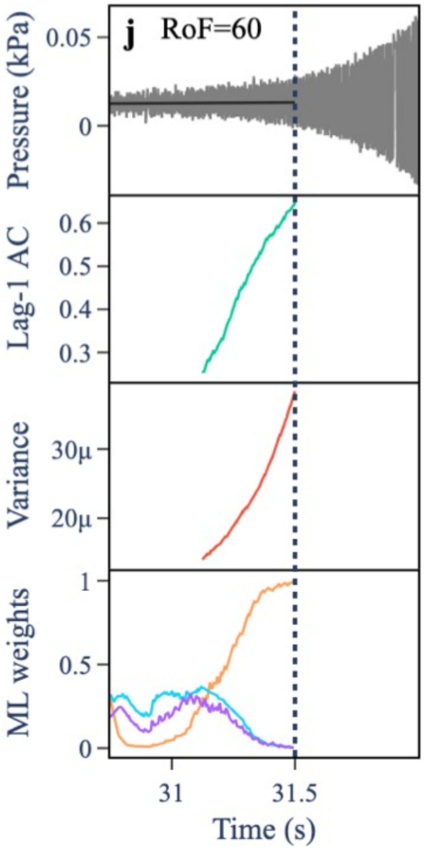

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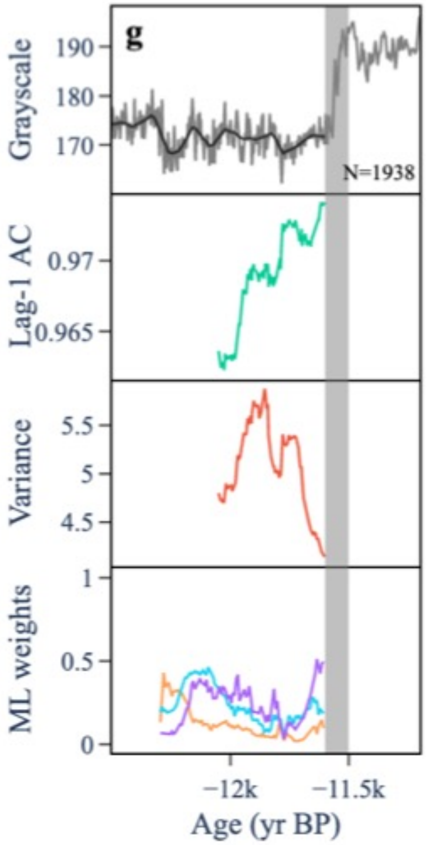

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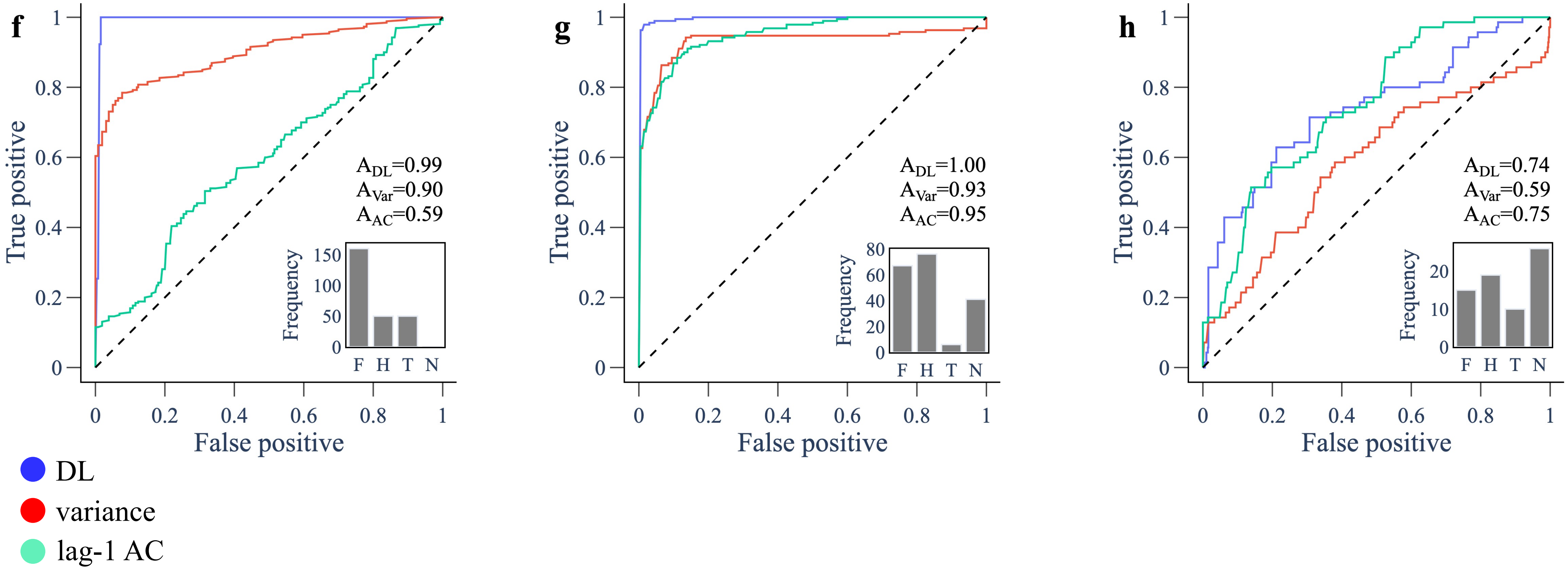

Conclusions

In these study systems:

- Deep learning approach provides greater sensitivity and specificity than variance or lag-1 AC
- Deep learning approach can distinguish between incoming fold, Hopf and transcritical bifurcations

More broadly:

- Neural networks can be trained to identify generic features of bifurcations – as demonstrated by application to systems outside of training set

Extensions

- Other bifurcations. Here, restricted to local codimension-1 bifurcations in continuous-time systems
- Spatial systems

Bury, T., Sujith, R., Pavithran, I., Scheffer, M., Lenton, T., Anand, M., & Bauch, C. Deep learning for early warning signals of regime shifts. *bioRxiv*. (2021)

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