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On cooperative learning in undergraduate mathematics courses

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1 Introduction

Cooperative learning is becoming increasingly recognised as a powerful pedagogy, capable of transforming the way students learn. Numerous studies have documented its success, and it is backed by a deep historical research base in the psychology of learning. Its use in undergraduate mathematics courses however, remains elusive. This article is an attempt to parse the underlying reasons for this resistance to change, and provides reasons for why mathematics-based courses can also benefit from a shift in the direction of cooperative teaching strategies. We complement our reasoning with a literature review of the recent studies in the field that have applied / analysed cooperative learning as a pedagogy. To encourage the use of this technique to fellow teachers of mathematics, we provide two implementation strategies that we believe are most conducive to the undergraduate mathematics classroom. Finally, we discuss the possible challenges that may dissuade mathematics instructors from leaving the comfort zone of traditional lecturing, and suggest techniques to overcome such challenges.

1.1 Cooperative vs. collaborative learning

Cooperative and collaborative learning are often used synonymously. However we adopt the convention that collaborative learning refers to *any* situation where groups of students work together. Cooperative learning is a *type* of collaborative learning whereby students all work towards a common goal, and are held individually accountable for their own work (Faust & Paulson, 1998). In the well-reputed review of cooperative learning 'across the disciplines', Millis sums it up as follows:

> *Cooperative learning is a highly structured form of group work that focuses on the problem solving that - when directed by an effective teacher - can lead to deep learning, critical thinking, and genuine paradigm shifts in students' thinking. Two givens...are positive interdependence and individual accountability.*
>
> (Millis, 2010, p. 5)

The latter two properties set apart cooperative learning from other forms of group work. When an instructor creates *positive interdependence*, students are incentivised to work with each other; going alone does not bode well for cooperative learning activities. Positive interdependence can be designed into the activity, for example by posing challenges that a single student could not manage on their own. *Individual accountability* ensures that students do 'ride-off the back' of each other. This behaviour can damage the learning experience, and so it is very important (particularly if grades are involved) that the task is designed such that students *earn* the marks they are given.

One can go further and categorise cooperative learning into three distinct types (Johnson, Johnson, & Smith, 1998). They are as follows:

- **Informal cooperative**: Groups of students are put together on the spot, making them convenient for in-class work. A popular technique of informal cooperative learning is the 'Think-Pair-Share', which we elaborate on in Chapter 3.
- **Formal cooperative**: For tasks that require more time, potentially with a hand-in, one may adopt formal cooperative learning. Groups should be heterogeneous in terms of gender, ethnicity and academic performance. In this extended period of time the instructor should try to monitor student discussions and intervene if necessary.
- **Base group**: For long-term projects, one may use base groups that are formed at beginning of the course. This allows students to develop relationships within their groups.

In this article we focus primarily on informal cooperative learning, since it may be implemented with least invasion to traditional lecturing techniques and is thus the first step to breaking free from the dogma. But let us first consider why mathematics in particular has been hesitant to board the train towards cooperative learning? The following section discusses possible explanations.

1.2 Nuances of mathematics

Perhaps active learning techniques are rare in math classes due the the nature of mathematical material itself. We consider three typical features of undergraduate mathematics material that potentially are a cause for resistance to cooperative learning in the classroom.

- **Objectivity**: Mathematics is a relatively objective subject, particularly at the undergraduate level. At this stage, students spend time learning the foundations of mathematics which, aside from a philosophical standpoint, leave little room for debate. Originating from fundamental axioms, mathematical theory emerges from sequences of logical steps. Once a theorem has been proven, it is 'set in stone', no valid argument exists against it. This rigour potentially hinders the possibility for open discussion and certainly opinionated debate. A debate around the answer of a mathematical problem would end up with one side being wrong and the other right, which is not fun - there should always be two sides to a debate. However, we will see in Chapter 3 how an instructor can resolve this dilemma by using carefully constructed mathematical problems that, for example, admit multiple *approaches* to the correct solution or can be viewed from different perspectives. Problems of this form can generate exciting discussion.

- **Sequential**: Mathematics builds upon itself. A true understanding of a mathematical concept requires an understanding of the mathematics that it is built upon - layer upon layer. It is therefore imperative that math syllabi are organised in a way such that material does not appear before the necessary pre-requisites. What's more, this stresses the importance for lecturers to cover all the material in the syllabus, exerting a time pressure. Undergraduate mathematics courses do not have the luxury of cutting syllabus material to provide extra time for cooperative learning activities. We discuss ways of overcoming time pressures in Chapter 4.

- **Abstract**: For most, learning mathematics well takes time and deep, undistracted thought. This is in part due to the abstract nature of the material. Yoon, Kensington-Miller, Sneddon, and Bartholomew (2011) conducted interviews to find that the majority of their math students did not fully understand their lectures immediately and needed time on their own to go through the notes with careful thought. Jumping straight into cooperative learning activities upon first sight of new material can be overwhelming for students that are used to learning the material carefuly in their own time. These activities also require the ability to discuss material in abstract terms, a vital skill, however something potentially off-putting for inexperienced students, making cooperative learning difficult to implement. This issue is also addressed in Chapter 4.

These characteristic features of mathematical material may indeed contribute to the resistance of active learning in this field. However, we argue that in many ways, cooperative learning is compatible if not harmonious with these features. For example, the abstract nature of mathematics may dissuade students from discussing it verbally, however given the importance of this skill in research and industry, what better way to practise this skill than through cooperative learning exercises? Objective material too, can provide exciting discussion - the success of which relies upon the careful construction and implementation of the discussion topic. Both are investigated in this article to provide helpful advice to those considering this rewarding teaching technique in a math-based class.

1.3 Breaking the norm

Recall the expression 'old habits die hard'. Teaching strategies are no stranger to this concept, that once a certain style has been adopted for many years it is very difficult to break free - even if there is evidence to suggest alternate styles are more effective. Lectures have been the dominant undergraduate teaching method in mathematics for centuries, why change now? Moreover, there is an expectation grounded in undergraduate math students that they should sit passively and not disrupt the lecture (Yoon et al., 2011). So not only does the norm among mathematics instructors need to be broken - but that among the students too. In the same study, the authors found that students overall appreciated the opportunities to interact with each other, despite initially putting them out of their comfort zones. The general consensus among the literature seems to suggest that breaking the norm is the biggest challenge - from then on cooperative learning becomes a natural part of the course from which extensive learning benefits arise. In order to break a norm, motivation for change must cross some threshold.This threshold in mathematics lecturing has not been crossed, but motivation for change continues to grow.

2 Motivation for change

There are an increasing number of studies demonstrating the learning benefits for students that participate actively in the classroom (Faust & Paulson, 1998). Here we provide a concise review of these benefits in a mathematical setting, along with the relevant studies to back them. We partition these benefits into five broad categories, although it should be noted that there is much overlap between the two.

2.1 Psychological / Well-being

Studies have show that cooperative learning activities can enhance students' social and psychological well-being (Johnson et al., 1998), which should come as no surprise. Cooperative learning facilitates an environment for students to form healthy relationships with their peers. Rosenthal (1995) documented a carry-over effect in their probability class whereby students would become comfortable working with each other in-class and thus continue to discuss course material outside of class hours. This bonding not only improves student achievement, but provides a supportive environment for studying undergraduate courses which can be overwhelming, particularly for students transitioning from high school. Math-induced anxiety is also not uncommon for undergraduate students and can be detrimental to student performance and well-being. Student responses to cooperative learning in engineering mathematics classes conducted by Cavanagh (2011) suggested that many found the activities to increase their confidence in the subject, since they are exposed to more than just the correct answer from the instructor, but a wide range of different answers coming from other students. One student is quoted as saying '[Cooperative learning activities] have also helped me break free from my fear of maths'. Given today's figures on student anxiety and depression, steps that can be taken within the classroom to improve student well-being should be considered of utmost importance.

2.2 Cognitive

Human brains are certainly not homogenous among individuals - people learn in different ways. Why should students be subjected to one style of teaching with this in mind? Incorporating different teaching styles makes sure we don't discriminate against groups of students that learn best in certain ways. Cooperative learning allows students to exercise parts of the brain that correspond to verbal, interactive performance.

2.3 Motivational

Some psychologists claim that the typical attention span of undergraduate students is about 15 minutes, and yet many instructors will lecture for up to 90 minutes non-stop. This does not allow students to stop, think, and act on the material they have just received, and important part of information retention. (Young, Robinson, & Alberts, 2009) found that breaking up the monotony of a lecture with short breaks for cooperative exercises, helped students to re-engage with the content when they switched back to lecturing. Once students become lost, they can readily become demotivated - cooperative learning helps retain motivation by addressing misconceptions and keeping students engaged (Machemer & Crawford, 2007). More generally a meta-analysis of 383 published reports on small-group learning in STEM subjects showed that this style of teaching promotes greater achievement, retention and favourable attitudes toward course material. (Springer, Stanne, & Donovan, 1999). When students know they will be active in class instead of sitting passively, they should also be more motivated to *prepare* for class - something that is highly beneficial to student learning.

2.4 Interpersonal

The development of interpersonal skills does not take place within a traditional mathematics degree. This needs to change. Interpersonal skills are vital for future career prospects of graduating mathematicians, regardless of whether they stay or leave academia. Getting students to discuss their work with peers and cooperate in problem solving develops this skill. Facilitating interaction between students and instructor also encourages students to critique the ideas that have been presented, allowing the lecturer to elaborate on certain areas that were not fully understood. During non-stop lecturing, many students avoid asking questions for fear of disrupting the class, or their question being trivial. Smith, Sheppard, Johnson, and Johnson (2005) also found that critical thinking skills improved when student participation was involved. Cooperative learning exposes students to different perspectives on the same mathematical problem that their peers may have, giving them a richer viewpoint of the problem. A student from the class of Cavanagh (2011) is quoted as saying 'hearing other class mates' responses opens different perspectives'. Students are also more likely to participate in active discussion with the class if they cooperate in small groups first, where there is less pressure to make a mistake.

2.5 Societal

In the grand scheme of things, incorporating cooperative learning during the early stages of professional development will prepare students to go on and tackle the challenges that society will face in the coming century. Mathematics is becoming ever more valuable in the work environment, be it understanding the behaviour of complex systems with mathematical models, or designing the latest cybersecurity to protect our sensitive information from hackers. Those with a mathematical background play an important role in society, and to play this role well, communication skills are vital. These skills should thus be incorporated in the undergraduate mathematics learning process, and cooperative learning provides just that. Exposing students to information in a variety of ways (rather than straight lecturing) better prepares them for the diversity of the working world.

As well as benefiting the student learning experience, note that cooperative learning is also a beneficial exercise for the instructor - it gives them the opportunity to be the *listener*. Letting the students do the talking gives the instructor direct feedback on student conceptions (and misconceptions) of the course material, which the instructor can then use to tailor their explanations. Moreover, having the students critique the ideas given in the lecture allows the instructor to explain things in different ways to accommodate each students way of thinking. Being able to articulate concepts in different ways is an important skill for a lecturer that is practised with the use of interactive discussion with the class.

3 Cooperative learning activities

There are a host of different ways to incorporate cooperative learning into the classroom. For a review of current methods, we refer the reader to Faust and Paulson (1998). We consider two of these methods, which we believe are most suited to mathematics-based courses with a large attendance, namely the Think-Pair-Share method and small group activities. We hope that they provide a starting point for instructors in this field, to experiment with cooperative learning, and subsequently tailor it to suit their circumstances.

3.1 Think-Pair-Share

Possibly the most popular form of cooperative learning is that of the 'Think-Pair-Share' (TPS) and for good reason. The TPS allows students to interact with each other without absorbing too much of lecture time. It shares many of the advantages of group work without the hassle of having to organise groups (Faust & Paulson, 1998). The instructor begins by posing a thought-provoking question to the class. Constructing an appropriate question is essential and discussed below in more detail. The class is then given a couple of minutes to *think* about the question and jot down any ideas. Following this, students *pair*-up and discuss their ideas, promoting peer-to-peer instruction and cooperative learning. It's useful if the instructor can circulate the class to prompt student pairs that are struggling. The shyer students are also more likely to ask the instructor questions in this time since they do not have to address the entire class. Finally the instructor gives the opportunity for students to *share* their thoughts with the entire class and facilitates a classroom discussion.

On implementing the TPS for the first time in the course, it is important to outline the procedure and expectations of the method and discuss 'the point' of active learning. Be explicit in the instructions to avoid any confusion - if students don't understand the procedure, their discussion is more likely to be about their weekend plans. Some students may initially be hesitant to break their passive role, but studies have found that the proportion of students who appreciate this engagement is high and increases over the course of the term as they become more comfortable with their peers (e.g. Ortiz-Robinson & Ellington, 2009). The instructor may even wish to discuss the studies in psychology that document the cognitive benefits of active learning.

Constructing an effective question

The key to the success of this method lies in the question / challenge that is presented to the students. We suggest three important features to consider: difficulty, novelty and applicability (this could be referred to as the DNA of the question!).

- **Difficulty**: A question needs to be challenging enough so that even the brightest students are kept pondering during the *think* period. Nonetheless, the question should be comprehensible else students will quickly lose motivation to approach it at all. Striking a perfect balance requires insight into the students capabilities, which may take a few trial runs to establish. It is the hope that students critical thinking skills improve over the term as a result of these activities, so starting off light and working up to harder problems as the term progresses is a natural approach.

- **Novelty**: The question should be exciting and thought-provoking and so instructors should avoid questions of a form that students have seen before on previous assignments, for example. Novelty is important for generating discussion. Note however that novel questions should still be closely related to the lecture material - just in a way that the students haven't seen before. The TPS may be used as a prelude to lecture content by getting students to think about a concept before they are formally taught it. Alternatively the TPS could serve as a way to think more deeply about a concept that has just been taught. The two approaches develop different thinking skills - the creative and the explorative respectively, which were both used to great success by Cavanagh (2011).

- **Applicability**: Most students transitioning from high school mathematics will not have seen the wide applicability of math to real-world phenomena. Cooperative learning is a good opportunity to put mathematics into perspective, by providing challenges that directly apply the theory. Chang (2011) provide many such examples for an undergraduate linear algebra course. Seeing the applicability of the course material not only motivates students to learn the theory, but also helps them to develop the important skill of *interpreting* mathematical results. In the applied setting one does not simply solve equations, we must learn to formulate them from physical justification, and interpret the results they give. In the setting of pure maths, where motivation is contained within the mathematical realm itself, applicability of the problem refers to a question being applicable to further components of the theory itself.

The TPS question may itself be motivated by something prior in the lecture. Cavanagh (2011) found playing a short video relevant to the problem to serve as a good catalyst for the TPS.

3.2 Think-Aloud Pair Problem Solving

An alternative technique, developed more recently by Brent and Felder (2016) gives specific roles to the students and implements peer-to-peer teaching more directly than the TPS. Going by the name Think-Aloud Pair Problem Solving (TAPPS), this approach involves students working together in pairs with a handout that contains both course material and problem solving activities. In this way, instructors are able to cover course content at the same time as implementing active learning, making this technique appealing for syllabus-heavy courses that require the development of problem-solving skills. This teaching strategy is implemented as follows:

1. **Prepare class handout:** The class handout is typically more involved than a TPS problem. It could include key concepts of course material with blanks for the students to fill in, as well as broader questions for discussion. The handout should still involve some form of problem solving, that the students work together towards. The handout

is designed to be informative, as well as a challenge, so it also serves as a reference for students later on in the course. The instructor could refer to the handout several times throughout the class.

2. **Organise students into pairs and their respective roles:** Ask students to designate themselves as either member A or B. Student A takes the role of the *explainer*, who talks about the descriptive part of the handout, filling in any blanks, and explaining why specific formulas and methods have been chosen. Student B, the *questioner*, keeps the explainer on their feet by questioning anything they feel is incorrect or unclear, and gives hints if they know something the explainer does not. Allow a short time (1-3 minutes) for this activity to take place. The next time students refer to the handout, their roles are reversed.

3. **Generate classroom response:** Randomly call upon students to share their responses with the class, making sure to be supportive if they are struggling. Where students give correct answers, write them up on the board so the entire class can copy down a correct solution. Make sure to draw responses from many different students to get a range of answers. This should allow for misconceptions to be addressed, and exposes students to various mistakes that can be made and how to avoid them. Emphasise that being wrong is the best way to learn.

This technique can be used on and off throughout the class, providing students with a healthy balance of passive and active learning. Unlike the TPS, the TAPPS involves a handout that serves not only problem solving, but as a reference for course material. Students are also given more specific instructions with regards to their roles, which could also facilitate more productive interaction between the pairs. Adopting the 'questioner-explainer' framework should help to balance the conversations between the more and less talkative students.

4 Overcoming Implementation Challenges

Despite the overwhelming success had by the majority of instructors who have incorporated segments of cooperative learning into their teaching, it seems many mathematics instructors are hesitant to break the continuous lecture. In this section we outline some of the perceived drawbacks to cooperative learning, and suggest ways to overcome these.

- **Less material covered**: Evidence suggests that comprehension and retention of information is higher with student participation (Springer et al., 1999), and so many subjects agree to bargain quantity for quality. However mathematics instructors have the difficulty of a large syllabus that cannot be restricted without jeprodisring students ability to take subsequent courses. A number of approaches can be taken to allow for cooperative learning and the completion of a large syllabus. Rosenthal (1995) constructed group exercises that covered material that lecture time would have otherwise - for example using a group exercise to review a topic meant that this did not have to be reviewed so extensively in later lectures. Alternatively students can be held responsible for covering some of the material in their own time. This material can then be used in the cooperative learning exercises, to ensure it has been understood. The extreme variant of this is the flipped classroom, which though rare in a mathematical setting, has been used with success in a calculus course (Ziegelmeier & Topaz, 2015).

- **Increased class preparation time**: First-time around, preparing for a cooperative learning activity takes a little extra time - mainly to construct an appropriate question to get the class thinking. However, just like lecture material, these activities can be used year upon year with little extra effort. The main hurdle for instructors new to the idea is the development of teaching *habits* at which point one can jump into cooperative learning activities with no extra preparation time.

- **Maintaining a constructive environment**: Staging the activity in an positive light puts students are on your side with regards to trying something new. Build up to the more involved cooperative learning activities by starting with the low-risk strategies, a short TPS is an ideal starting point. As students become more comfortable with the activities, facilitating a constructive environment will become easier.

- **Having top students teach other members of the group**: Some argue it not fair that the stronger students end up teaching those that struggle more, and that creating heterogeneous groups brings down the students that work at a faster pace. However, most teachers will agree that the act of teaching greatly enhances one's one learning. In fact, if you find yourself not able to teach something that you thought you understood, perhaps you don't understand it well enough. The top students will develop important skills in explaining their understanding as opposed to simply convincing themselves of their understanding. Without cooperative learning activities, even the hardest working students aren't developing this skill set.

- **Student apprehension in engaging with unfamiliar peers**: Some students may feel uncomfortable conversing with those they don't know, however this is unlikely to last if cooperative learning continues to be implemented. As students continue to meet others in the course, the working environment should become more familiar and supportive. A student from (Cavanagh, 2011) stated 'sometimes just to talk to the person next to me, though uncomfortable at first, allowed me to lift the level of my own learning'. The instructor can make the initial transition to cooperative learning easier for students by explicitly encouraging students to introduce themselves or even using an icebreaker to reduce the perceived formality of the lecture.

- **Student resistance to active learning**: It is suggested (Weimer, 2012) that students who resist active learning generally do so for one of the following reasons: 1) it requires them to do more work, 2) they feel threatened by the greater responsibility 3) they feel less productive working with others, or 4) they feel they lack the intellectual maturity to perform as expected. However over time this is unlikely to persist. Ortiz-Robinson and Ellington (2009) experimented with cooperative learning techniques in a course of Real Analysis - a domain of pure mathematics that is typically lectured through and through. Students indeed showed an initial resistance to the activities, with only 50% of them considering the cooperative activities as beneficial to their learning. However towards the end of the course this figure had risen to 78%, demonstrating a vast increase as students became more comfortable with this style of learning and the benefits of cooperative learning kicked in.

Thus, none of these implementation challenges should pose as a significant obstacle to the incorporation of some form of cooperative learning in the undergraduate mathematics classroom. It is simply a matter of instructors finding the will to break out of the comfort zone that is lecturing and setting some time aside to construct relevant activity questions. Given the vast educational benefit of engaging students through cooperative learning, there is little excuse to not a least give it a try. An old proverb sums up nicely:

I hear and I forget,
I see and I remember,
I do and I understand.

5 Conclusions

Cooperative learning has found its rightful place across many academic disciplines, however it remains elusive in the mathematical domain. This paper has addressed possible reasons for this resistance but submits that these reasons are outweighed by the merits of student success and satisfaction when exposed to cooperative learning activities. We have provided the reader with two effective cooperative learning strategies with minimal class disruption and preparation time. These serve as a good starting point, however we expect that in time, mathematics teachers will develop personalised cooperative learning methods that are best suited to their teaching style. We hope that future discussion around mathematics teaching will address not *whether* to include cooperative learning in the classroom, but *how* to best incorporate it into the currently lecture-heavy discipline.

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