Lecture 5.

Announcements:

- lecture notes on Learn
- tutorial - overview of common functions and graphs.

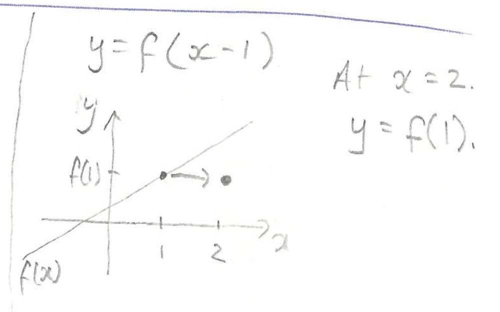
Today's topics:

- Function transformations
  - shifting, reflecting, scaling

  Ch 2.2.1
  Example 2.7
  EoL 5.

- modelling using functions

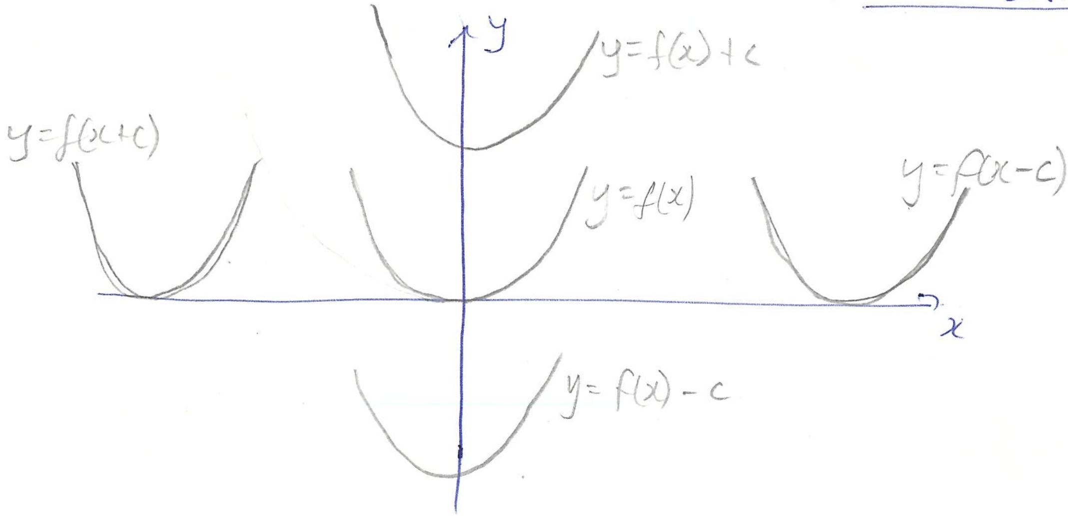
Vertical / Horizontal Shifting.



Suppose $c>0$.

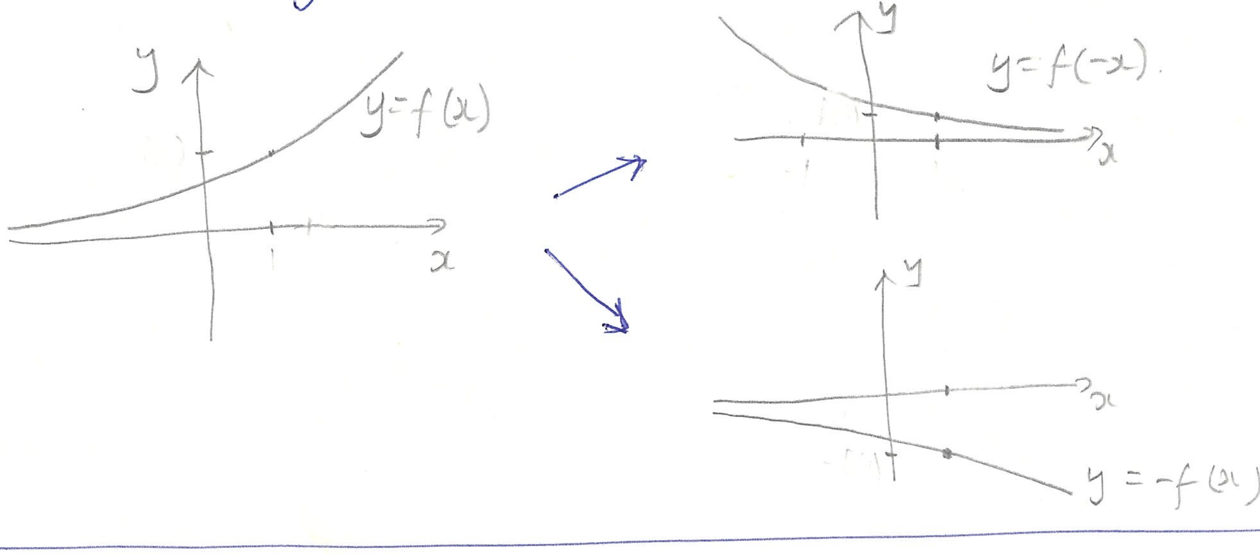
1. $y=f(x)+c$: shift up by c units
2. $y=f(x)-c$: shift down by c units
3. $y=f(x+c)$: shift left by c units
4. $y=f(x-c)$: shift right by c units

These transformations are known as translations.



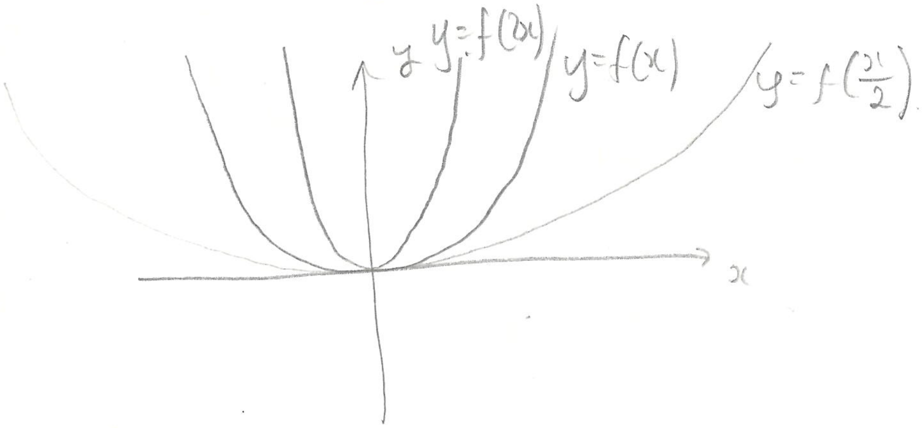
Reflections

1. $y = f(-x)$: reflection across y-axis.
2. $y = -f(x)$: reflection across x-axis.



Vertical / Horizontal Scaling

1. $y = f\left(\frac{x}{c}\right)$ horizontal stretch by a factor c. ~~horiza~~
2. $y = f(cx)$ horizontal squish by a factor c.
3. $y = cf(x)$ vertical stretch by factor c.
4. $y = \frac{1}{c}f(x)$ vertical squish by factor c.



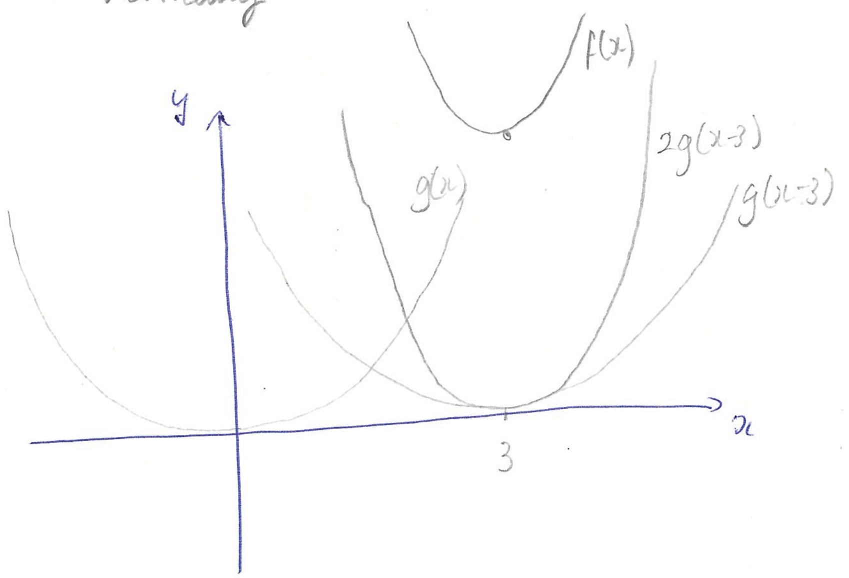
Example.

Sketch $f(x) = 2(x-3)^2 + 4$

Let $g(x) = x^2$

$f(x) = 2\,g(x-3) + 4$

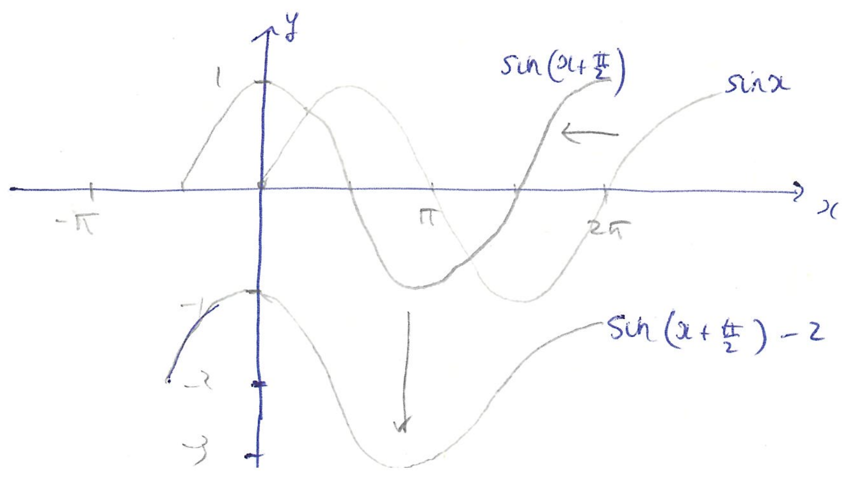
stretch vertically — shift right — shift up



Check answer with values e.g. $f(3) = 4$,
$f(4) = 6$
$f(2) = 6$ etc.

Example.

$f(x) = \sin\left(x + \frac{\pi}{2}\right) - 2$



Functions in science

In science, we use functions to model relationships between variables.

Eg/. Newton's law of gravitation

$$\underset{\text{force}}{F(d)} = \frac{Gm_1m_2}{d^2}$$

← constant

– relationship btw force and distance

← distance

Eg/ Diameter of tree over time (model)

$$\underset{\text{diameter}}{D(t)} = a\sqrt{\underset{\text{time}}{t}+b}$$

← constants

Terminology

~~"proportional to"~~

constant of proportionality

1. If $y = kx$ for some constant k, say y is <u>proportional to</u> x, written $y \propto x$.

Eg/. NLG, $F \propto \frac{1}{d^2}$ where $k = Gm_1m_2$.

2. Linear model : $y = ax + b$

Quadratic model : $y = ax^2 + bx + c$. etc.

3. Inversely proportional: $y \propto \frac{1}{x}$.

Fitting a model (function)

Tree model

parameters to 'fit'.

$$D(t) = a\sqrt{t+b}$$

Data

$D(0) = 40\text{ cm}$ - at time $t=0$, diameter $D=40\text{cm}$.

$D(20) = 60\text{ cm}$ - at time $t=20$ yrs, $D=60\text{ cm}$.

Given data, find a and b by plugging data into model.

$$\left.\begin{aligned} D(0) &= a\sqrt{b} = 40 \quad \text{.... (1)} \\ D(20) &= a\sqrt{20+b} = 60 \quad \text{... (2)} \end{aligned}\right\}$$

2 eqns
2 unknowns.
- can solve.

Method: find a as a function of b in (1).
then sub into ~~other~~ eqn (2).

From (1), $a = \dfrac{40}{\sqrt{b}}$.

Sub into (2).

$$\frac{40}{\sqrt{b}}\sqrt{20+b} = 60$$

$$\Rightarrow \quad 40\sqrt{20+b} = 60\sqrt{b}$$

$$\Rightarrow \quad 2\sqrt{20+b} = 3\sqrt{b}$$

$$\Rightarrow \quad 4(20+b) = 9b$$

$$\Rightarrow \quad 80+4b = 9b$$

$$\Rightarrow \quad 5b = 80$$

$$\Rightarrow \quad b = 16, \qquad a = \frac{40}{\sqrt{16}} = 10.$$

Model is

$$D(t) = 10\sqrt{t+16}.$$

check algebra:

$D(0) = 10\sqrt{16} = 40$ ✓

$D(20) = 10\sqrt{36} = 60$ ✓

Follow up qns.

1. If $t=0$ corresponds to 1997, when did tree start growing?

i.e. when was $D=0$?

plug into model: $0 = 10\sqrt{t+16} \Rightarrow t=-16.$

Started growing in ~~the~~ $1997-16 = 1981$..

2. In what year does diameter reach 70cm?

$70 = 10\sqrt{t+16}$

$\Rightarrow \sqrt{t+16} = 7$

$\Rightarrow t+16 = 49$

$\Rightarrow t = 33.$ year $1997+33 = 2030.$