Lecture 8

Tutorial 5.30 - trig

Read 1.3

Examples 1.36 - 1.37

Ex. 1.3.1 - 1.3.4

1.3.10, 1.3.11*

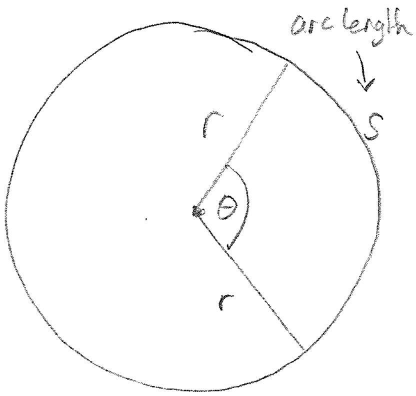
Today's topics:

- trigonometry (part I)

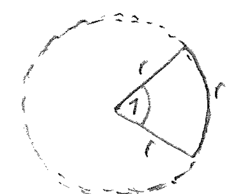
Angles

arbitrary ↓ arises naturally ↓

- Full circle has $360° = 2\pi$ radians



- We use <u>radians</u> in calculus.
- At 1 radian, $s = r$ (definition of rad)
- It follows, that $s = r\theta$



- Full circle, $s = 2\pi r$ (circumference)

$\Rightarrow \theta = 2\pi$ radians.

Converting Degrees ⟺ Radians.

2π radians $= 360$ degrees $\qquad (\pi = 180°$ useful$)$

$\Rightarrow 1$ radian $= \dfrac{360}{2\pi}$ degrees

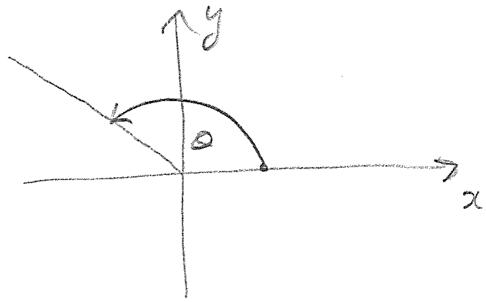
Eg/ 3π radians $= 3\pi \left(\dfrac{360}{2\pi}\right)$ degrees

$= 540°$

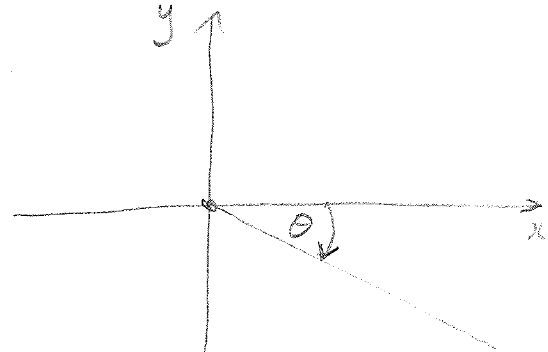
Eg/ $45° = \left(\dfrac{2\pi}{360}\right) 45$ rad $= \dfrac{\pi}{4}$ rad

Angle conventions

In x-y plane, measure from positive x-axis.

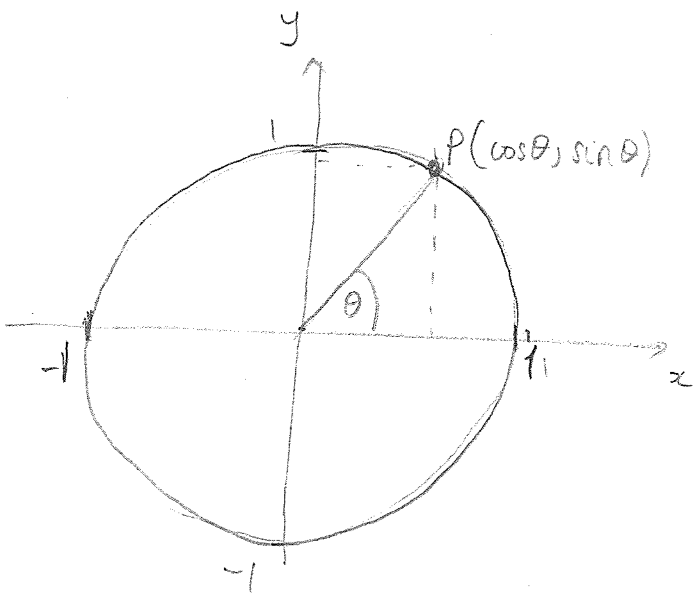


rotate counter-clockwise for $\theta > 0$.



rotate clockwise for $\theta < 0$

Definition of sine and cosine functions



Take unit circle

$\cos\theta$ - x-coord of P

$\sin\theta$ - y-coord of P

Eg/ $\cos(0) = 1$, $\sin(0) = 0$

$\cos\left(\frac{3\pi}{2}\right) = 0$, $\sin\left(\frac{3\pi}{2}\right) = -1$

Properties

Domain $= \mathbb{R}$

Range $= [-1, 1]$.

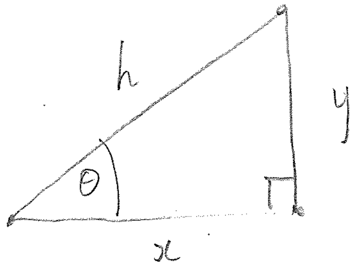
Symmetry: $\sin(-\theta) = -\sin\theta$ (odd)

$\cos(-\theta) = \cos\theta$. (even)

Periodicity: $\sin(\theta + 2n\pi) = \sin\theta$, $n \in \mathbb{Z}$

$\cos(\theta + 2n\pi) = \cos\theta$, $n \in \mathbb{Z}$. $(p = 2\pi)$.

Using Triangles (valid for $0 \leq \theta \leq \frac{\pi}{2}$)

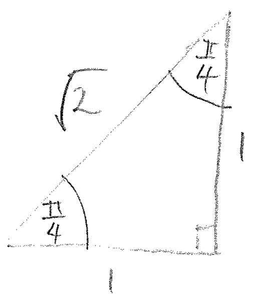


$$\sin(\theta) = \frac{y}{h}$$

$$\cos(\theta) = \frac{x}{h}$$

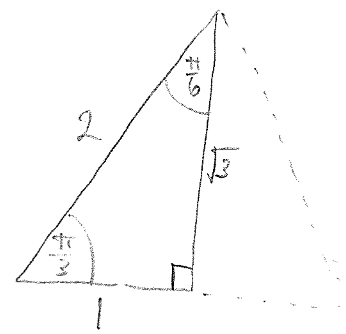
$$\tan(\theta) = \frac{y}{x} = \frac{\sin\theta}{\cos\theta}$$

Special Angles



$$\sin\left(\tfrac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\cos\left(\tfrac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$



$$\sin\left(\tfrac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

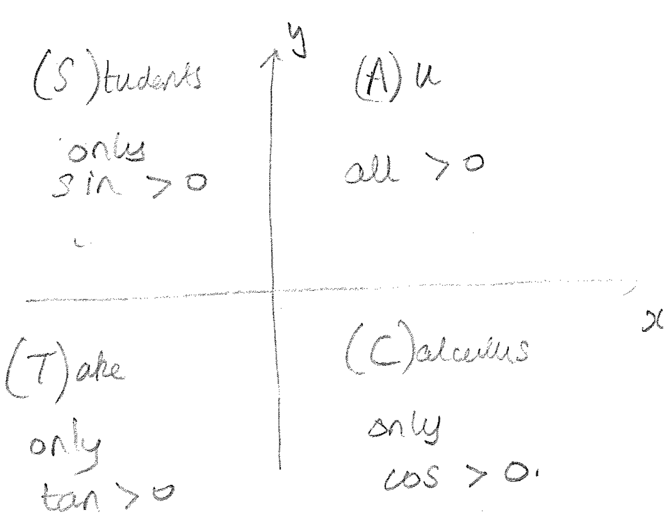
$$\sin\left(\tfrac{\pi}{6}\right) = \frac{1}{2}$$

$$\cos\left(\tfrac{\pi}{3}\right) = \frac{1}{2}$$

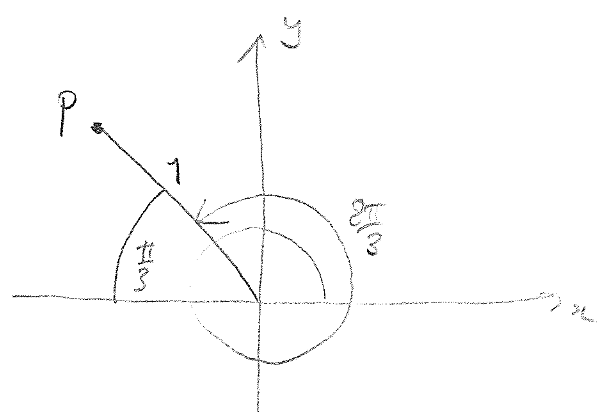
$$\cos\left(\tfrac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

Angles outside of $[0, \frac{\pi}{2}]$.

Use mnemonic



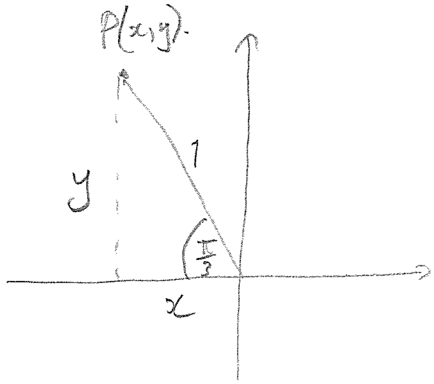
Eg/ Find $\sin\left(\frac{8\pi}{3}\right)$, $\cos\left(\frac{8\pi}{3}\right)$

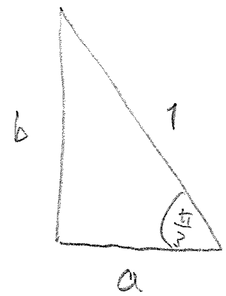


Find coords of P.

$\sin\left(\frac{8\pi}{3}\right) > 0$, $\cos\left(\frac{8\pi}{3}\right) < 0$.

Construct triangle (special angle)





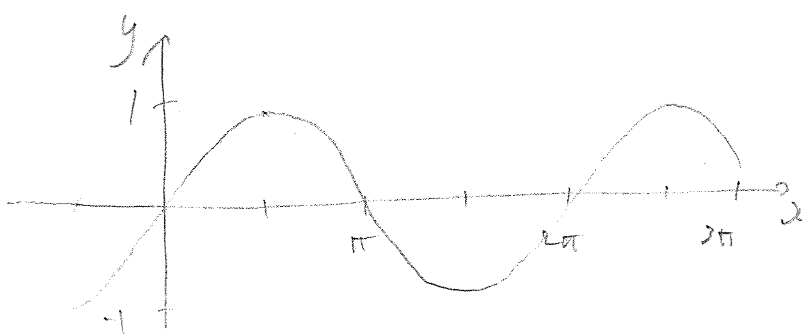
$$a = \cos\frac{\pi}{3} = \frac{1}{2}$$

$$b = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow x = -\frac{1}{2} = \cos\left(\frac{8\pi}{3}\right)$$

$$y = \frac{\sqrt{3}}{2} = \sin\left(\frac{8\pi}{3}\right)$$

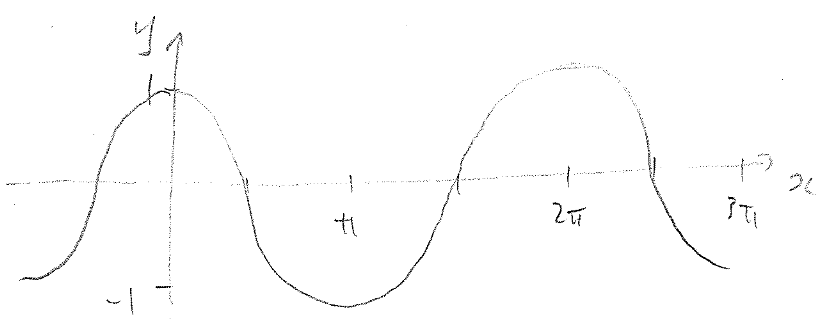
Graphs of trig functions



$y = \sin(x)$

$\mathbb{D} = \mathbb{R}$, $\mathbb{E} = [-1,1]$

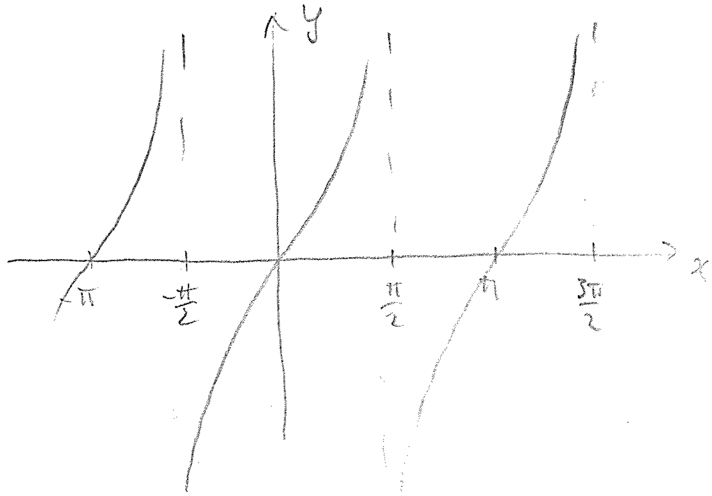
2π-periodic, odd



$y = \cos(x)$

$\mathbb{D} = \mathbb{R}$, $\mathbb{E} = [-1,1]$

2π-periodic, even



$$y = \tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\mathbb{D} = \left\{x \in \mathbb{R} : x \neq \frac{(2n+1)\pi}{2}, n \in \mathbb{Z}\right\}$$

$$\mathbb{E} = (-\infty, \infty) = \mathbb{R}.$$

π-periodic, odd.

Reciprocals of trig

"cosecant"

$$\csc(\theta) = \frac{1}{\sin\theta}$$

"secant"

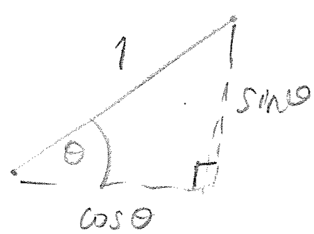
$$\sec(\theta) = \frac{1}{\cos\theta}$$

"cotangent"

$$\cot(\theta) = \frac{1}{\tan\theta}$$

Powers of trig

Notation: $[\sin(x)]^2 = \sin^2(x)$



Pythag: $\boxed{\sin^2\theta + \cos^2\theta = 1}$

$(\div \cos^2\theta)$ $\quad \tan^2\theta + 1 = \sec^2\theta$.