Lecture 9

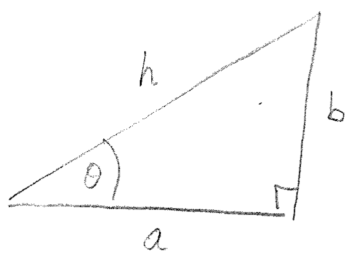
Today's topics:

- trigonometry (part II)
  - identities, modelling

Announcements:

- A1, EoL 2-7 due Friday.
- Quiz – Monday 1st Oct
  - 5.30pm
  - Trig (not inverses)

Trig Identities



$a^2+b^2=h^2$

$\sin\theta=\frac{b}{h}$, $\cos\theta=\frac{a}{h}$

$\sin^2\theta+\cos^2\theta=\left(\frac{b}{h}\right)^2+\left(\frac{a}{h}\right)^2=\frac{a^2+b^2}{h^2}=1$

$\Rightarrow \boxed{\sin^2\theta+\cos^2\theta=1}$

derive all other identities from here.
(Ch 1.3.5)

Compound angle formulae

$$\boxed{\begin{aligned}\sin(x+y)&=\sin x\cos y+\cos x\sin y\\ \cos(x+y)&=\cos x\cos y-\sin x\sin y\end{aligned}}$$

Then show (e.g.)

$$\sin\left(x+\frac{\pi}{2}\right)=\sin x\underbrace{\cos\frac{\pi}{2}}_{0}+\cos x\underbrace{\sin\frac{\pi}{2}}_{1}$$

$$=\cos x$$

$$\cos\left(x+\frac{\pi}{2}\right)=\cos x\cos\frac{\pi}{2}-\sin x\sin\frac{\pi}{2}$$

$$=-\sin x$$

Double-angle formulae

$$\sin(2x) = \sin(x+x) = 2\sin x\cos x$$

$$\cos(2x) = \cos^2 x - \sin^2 x$$

$$= 1 - 2\sin^2 x$$

$$= 2\cos^2 x - 1$$

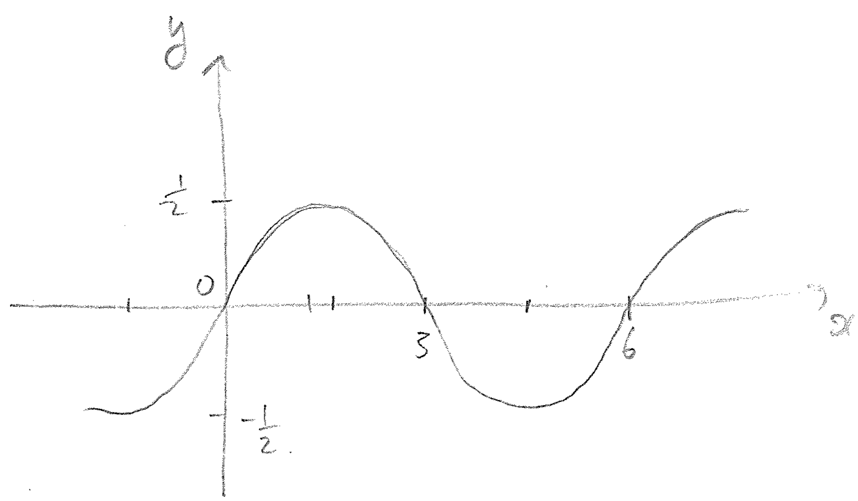
show using $\cos^2 x + \sin^2 x = 1$

Eg/. simplify & sketch

$$y = \sin\left(\frac{\pi x}{6}\right)\cos\left(\frac{\pi x}{6}\right).$$

Let $\theta = \frac{\pi x}{6}$.

$$y = \sin\theta\cos\theta = \frac{1}{2}\sin(2\theta) = \frac{1}{2}\sin\left(\frac{\pi x}{3}\right)$$



Sketch:

max $= \frac{1}{2}(1) = \frac{1}{2}$

min $= \frac{1}{2}(-1) = -\frac{1}{2}$.

period: $\frac{\pi x}{3} = 2\pi$

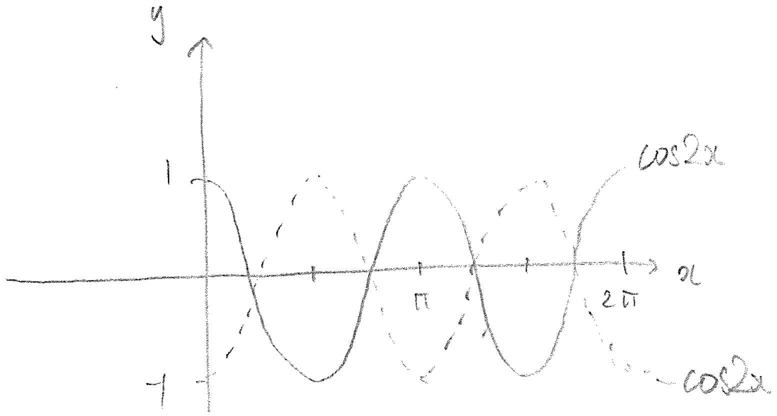
$\Rightarrow x = 6$.

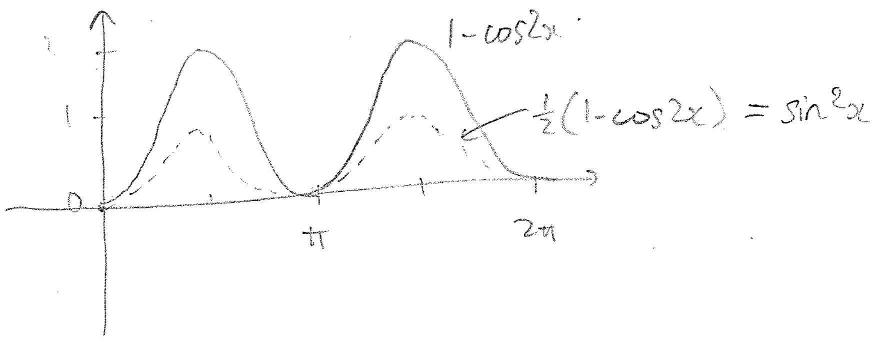
Eg/ Sketch $\sin^2(x)$.

Recall $\cos 2x = 1 - 2\sin^2 x$

$\Rightarrow \quad \sin^2 x = \frac{1}{2}(1 - \cos 2x)$

easier to sketch.





Modelling with trig

Mouse livers:

- mice forage & eat at night - liver larger.
- inactive during day - liver smaller.

8.00pm : end of resting, liver mass = 1.5g. (min)
8.00am : end of foraging, mass = 2.25g. (max)

Model using trig!

Building the model

Use framework

horizontal scaling determines period.

$$M(t) = A\cos(B(t + C)) + D$$

mass in g at time t hours.

vertical scaling (amplitude)

horizontal shift

vertical shift.

Range of $M(t)$

We know $-1 \leq \cos x \leq 1$

$\Rightarrow -A \leq A\cos x \leq A$

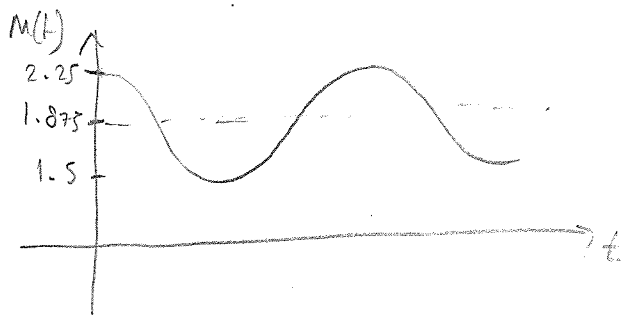
$\Rightarrow -A + D \leq A\cos x + D \leq A + D$

$\Rightarrow$ Range $= [D - A, D + A]$.

$\Rightarrow$ $\left.\begin{array}{l} D - A = 1.5g \\ D + A = 2.25g \end{array}\right\}$ solve: $D = 1.875g$, $A = 0.375g$.

So far:

$$M(t) = 0.0375 \cos(\ldots) + 1.875$$



Need to sort out timing. (Stuff inside the cosine).

Find the period

Period of liver oscillators = 24 h.

Period of $\cos Bt$:

$$Bt = 2\pi \Rightarrow t = \frac{2\pi}{B}.$$

Set $\frac{2\pi}{B} = 24h \Rightarrow B = \frac{\pi}{12} h^{-1}$.

Horizontal shift:

Let's start the clock at midnight ($t=0$ at 12.00am).

First max occurs at $t=8$ (8.00 am).

Max of $\cos Bt$ occurs at $t=0$. $\Rightarrow$ Shift right by 8.

$\Rightarrow C = -8$.

$$\Rightarrow M(t) = 0.0375 \cos\left(\frac{\pi}{12}(t-8)\right) + 1.875.$$

Sketch