Lecture 2.

Announcements.

- EoL 2-7 - due Sept 28.
- Tutorial 5.30 - cover pre-req. - $\boxed{\text{Ch 1.1}}$ → sets, intervals, inequalities
  STC 0010.

Learning objectives:-

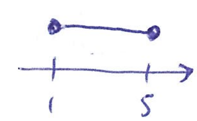
- use set notation
- find domain ~~and range~~ of a function.
- identify types and properties of functions.

What is a set?

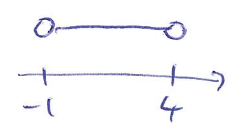
- collection of distinct objects ("elements") considered as a whole.

Eg/. $S_1 = \{1, 3, \ldots, a, b\}$

$S_2 = [1, 5]$ all real numbers between and including 1 and 5.



$S_3 = (-1, 4)$ real no.s between but NOT including -1 and 4



$S_4 = \{\text{tripod, skittles, footy}\}$

Set membership.

If element x belongs to set S, then write $x \in S$. If not, $x \notin S$.

Eg/. $3 \in [1,4]$.

$2 \notin (2, 5]$

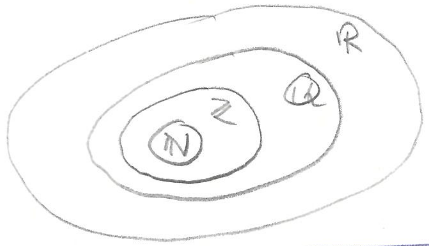
Important sets

$\mathbb{N}$ - natural numbers : $0, 1, 2, \ldots$

$\mathbb{Z}$ - integers : $\ldots, -2, -1, 0, 1, 2, \ldots$

$\mathbb{Q}$ - rationals : any ratio of integers.

$\mathbb{R}$ - real numbers : includes irrational numbers e.g. π, $\sqrt{2}$.



"a subset of"

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}.$$

Union and Intersection

Union: $X \cup Y$ includes elements that each belong to ~~belonging to either~~ X <u>or</u> Y

Intersection: $X \cap Y$ " ~~both~~ X <u>and</u> Y.

Eg/. $X = \{a, b, c\}$, $Y = \{b, d\}$.

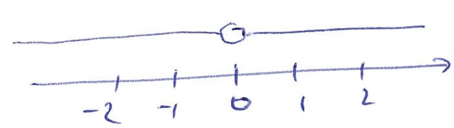
$X \cup Y = \{a, b, c, d\}$, $X \cap Y = \{b\}$.

Sets with conditions

* $:$ means "such that"

Eg/

$$\{x \in \mathbb{R} : x \neq 0\}.$$



$$= (-\infty, 0) \cup (0, \infty).$$

Eg/.

$$\mathbb{N} = \{x \in \mathbb{Z} : x \geq 0\}.$$

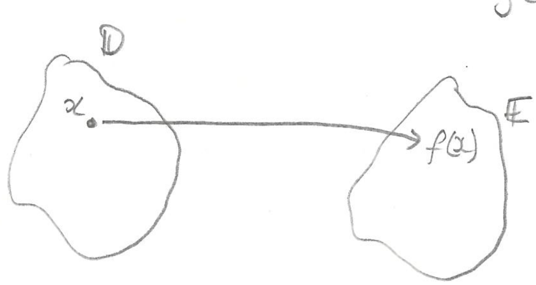
Practise. Ch 1.1.1 & Ch. 1.1.4.

Tutorial

What is a function?

A function maps elements from a given set to elements of another set.

domain $\mathbb{D}$ (given set); range $\mathbb{E}$ (another set)



Notation.

$$x \to f(x)$$

$$f : \mathbb{D} \to \mathbb{E}.$$

Each input in $\mathbb{D}$ is mapped to a specific output in $\mathbb{E}$.

Eg/ $\mathbb{D} = \{\text{tripod, skittles, footy}\}$.

$\mathbb{E} = \{1, 2, \dots, 10\} = \{x \in \mathbb{N} : 1 \leq x \leq 10\}$. "cuteness rating"

$f : \mathbb{D} \to \mathbb{E}$,

$$\left.\begin{aligned} f(\text{tripod}) &= 3 \\ f(\text{skittles}) &= 7 \\ f(\text{footy}) &= 7 \end{aligned}\right\} \text{ specific outputs.}$$

-(can't have $f(x) = 2$ or 4).

Eg/ Computer vision. $\mathbb{D} = \{\text{pixels in image}\}$, $\mathbb{E} = \{\text{cat, dog}\}$.

Eg/ $f(x) = x^2$.

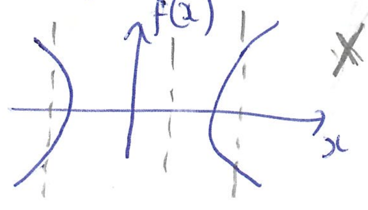
$\mathbb{D} = \mathbb{R}$, $\mathbb{E} = \{x \in \mathbb{R} : x \geq 0\}$

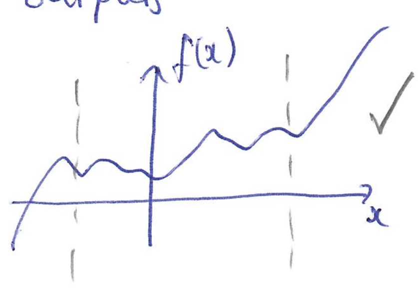
Vertical-line test.

$f(x)$ cannot have multiple outputs for same input.

vertical line should cross function at most once.

Eg/





Finding the domain of a function.

Function must be <u>defined</u> for all elements of domain.

Eg/. $f(x) = \frac{1}{x}$. $\mathbb{D} = \{x \in \mathbb{R} : x \neq 0\}$ (cannot divide by 0).

Eg/. $f(x) = \sqrt{x}$ $\mathbb{D} = \{x \in \mathbb{R} : x \geqslant 0\}$ (cannot take root of −ve #s).

Eg/ $f(x) = \frac{1}{\sqrt{4-x^2}}$

must have $4 - x^2 > 0$.

$\Rightarrow x^2 < 4$ $\quad\mathbb{D} = \{x \in \mathbb{R} : -2 < x < 2\}$.

$\Rightarrow -2 < x < 2$.

(Ex 2.1.1)

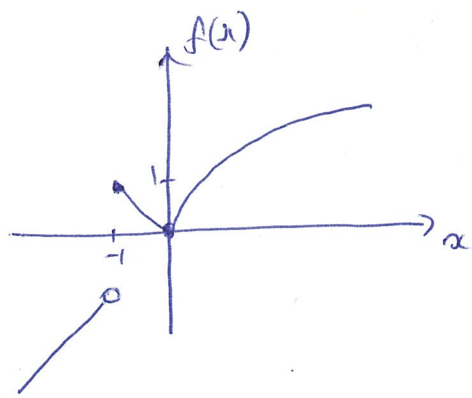
Piecewise Functions.

Functions can have different expressions on different parts of the domain.

Eg/.

$$f(x) = \begin{cases} \sqrt{x} & x \geqslant 0 \\ -x & -1 \leq x < 0 \\ x & x < -1 \end{cases}$$

$f(-2) = -2$, $f(4) = 2$ etc.



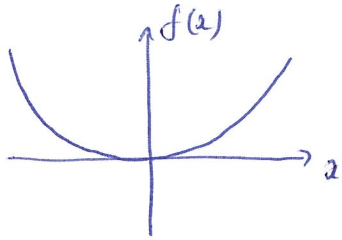
• include pt

∘ don't include pt.

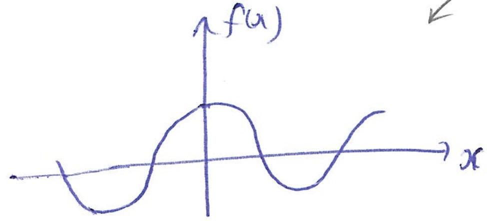
Even functions

Satisfy $f(x) = f(-x)$ for all $x \in \mathbb{D}$.

Eg/. $f(x) = x^2$.



$f(x) = \cos(x)$.

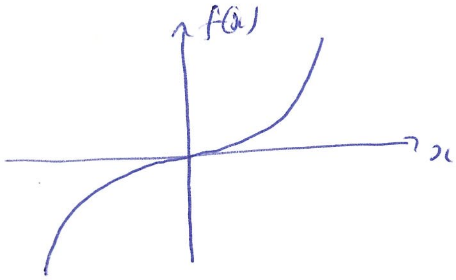


symmetric about y-axis.

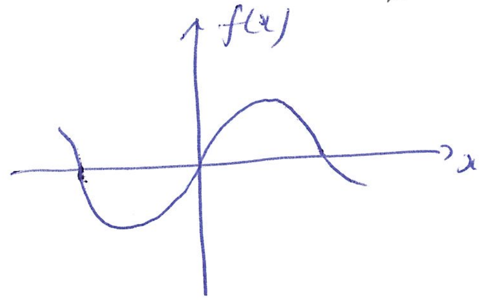
Odd functions

Satisfy $f(x) = -f(-x)$ for all $x \in \mathbb{D}$

Eg/. $f(x) = x^3$.

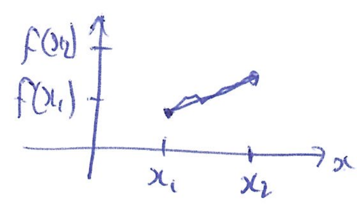


$f(x) = \sin x$.



rotational symmetry of 180° about origin.

Monotonic functions



~~Increasing function~~

Function increasing on interval if $f(x_2) > f(x_1)$ for all pairs $x_2 > x_1$ on interval.

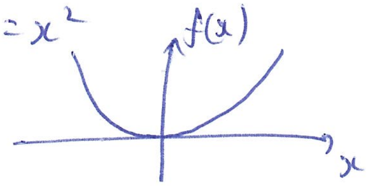
~~" decreasing~~

Similar for decreasing.

~~if f(x~~

f is monotonic if increasing or decreasing entirely on $\mathbb{D}$.

Eg/ $f(x) = x^2$



increasing on $[0, \infty)$
decreasing on $(\infty, 0]$.

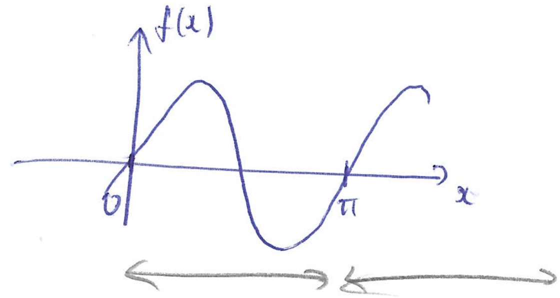
Periodic functions

with period p

Periodic if $f(x+p) = f(x)$ for all $x \in D$.

Period is smallest $p > 0$ such that this holds.

Eg/. $f(x) = \sin 2x$, $p = \pi$.



$$
\begin{aligned}
f(x+\pi) &= \sin(2(x+\pi)) \\
&= \sin(2x + 2\pi) \\
&= \sin 2x \\
&= f(x).
\end{aligned}
$$

Remember

EOL 2 - sequences covered next time.

Tutorial - sets, intervals, inequalities