Lecture 33.

Today's topics:

- Properties of definite integrals
- Indefinite integrals
- Net area vs total area

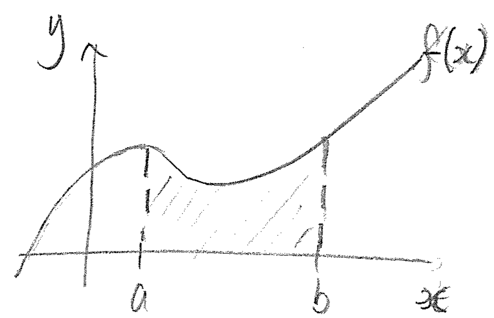
EoL 29, 31

Tutorial :- Riemann sum practise
- Project help

Review Ch 6.2, 6.3.

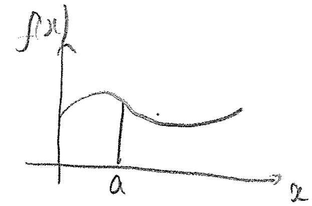
Last time: FTOC (ver II).

$$\underbrace{\int_a^b f(x)\,dx}_{\text{net area under } f \text{ from } a \text{ to } b} = \underbrace{F(b) - F(a)}_{F \text{ is an antiderivative of } f}$$



Useful properties of definite integrals.

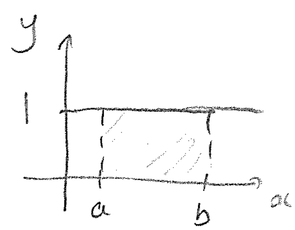
1.) $$\int_a^a f(x)\,dx = 0$$



Area of a region with zero width.

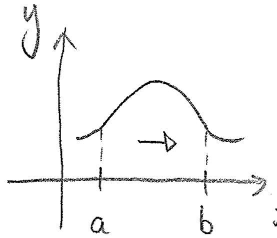
2) $$\int_a^b 1\,dx = b - a$$

$$\left(\int_a^b dx\right)$$

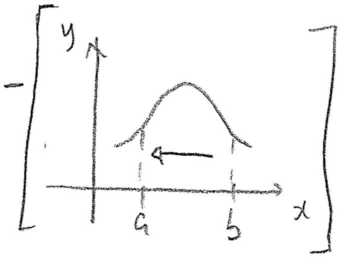


Rectangle of area $= b - a$.

3.) $$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$



$= -$



Recall in defn of integral

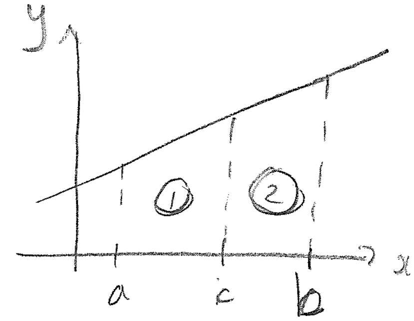
$$\Delta x = \frac{\text{upper}-\text{lower}}{n}$$

$\uparrow$ negative when upper bound < lower bound.

4.) $$\int_a^b c\,f(x)\,dx = c\int_a^b f(x)$$ ← Same for Σ, $\frac{d}{dx}$.

5.) $$\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx.$$

6) $$\int_a^b f(x)\,dx = \underbrace{\int_a^c f(x)\,dx}_{①} + \underbrace{\int_c^b f(x)\,dx}_{②}.$$



Examples:

a) $$\int_2^{-1} \sqrt{3}\, x^2\, dx = -\int_{-1}^{2} \sqrt{3}\, x^2\, dx \quad \text{(rule 3)}$$

$$= -\sqrt{3}\int_{-1}^{2} x^2\, dx \quad \text{(rule 4)}$$

$$= -\sqrt{3}\left[\tfrac{1}{3}x^3\right]_{-1}^{2}$$

$$= -\sqrt{3}\left(\tfrac{1}{3}8 - \tfrac{1}{3}(-1)\right)$$

$$= -\frac{\sqrt{3}}{3}(9) = -3\sqrt{3}.$$

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b) $$\int_0^{\pi} \left(\sin x + 2 + g(x)\right) dx \quad \text{where} \quad \int_{-\pi}^{\pi} g(x)\,dx = 2$$

$$\int_{-\pi}^{0} g(x)\,dx = 1$$

$$= \int_0^{\pi} (\sin x + 2)\,dx + \int_0^{\pi} g(x)\,dx \quad \text{(rule 5)}$$

$$= \left[-\cos x + 2x\right]_0^{\pi} + \int_{-\pi}^{\pi} g(x)\,dx - \int_{-\pi}^{0} g(x)\,dx. \quad \text{(rule 6)}$$

$$\left(\int_{-\pi}^{0} + \int_{0}^{\pi} = \int_{-\pi}^{\pi}\right)$$

$$= -\cos\pi + 2\pi - (-\cos(0) + 2(0)) + 2 - 1$$

$$= 1 + 2\pi - (-1) + 1$$

$$= 3 + 2\pi.$$

The Indefinite Integral.

→ Old concept, new symbol.

→ The indefinite integral of $f(x)$, denoted $\int f(x)\,dx$
the general antiderivative of f:

$$\int f(x)\,dx = F(x) + C.$$

no bounds.

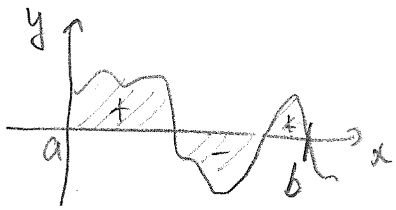
Eg, $\int \sqrt[4]{x^5}\,dx = \int x^{5/4}\,dx = \dfrac{x^{9/4}}{(9/4)} + C = \dfrac{4}{9}x^{9/4} + C.$

Note: $\int_a^b f(x)\,dx$ gives a single value

$\int f(x)\,dx$ gives a family of functions.

Net Area vs. Absolute Area.

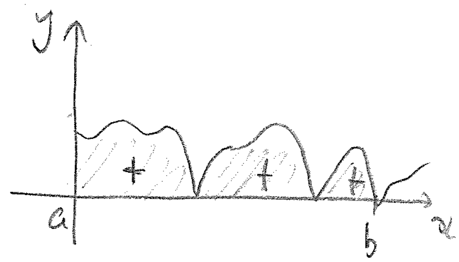
→ The integral gives the <u>net area</u> under the curve.



$$\int_a^b f(x)\,dx$$

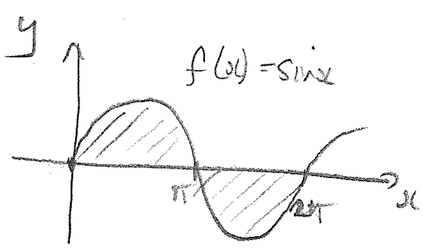
→ How do we compute the <u>absolute</u> area?

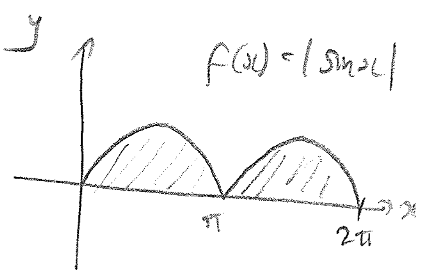
<u>Ans</u>: use absolute value of function.



$$\int_a^b |f(x)|\,dx.$$

Eg1. Find the <u>absolute area</u> under $y = \sin(x)$ from $x=0$ to $x=2\pi$.





$$\int_0^{2\pi} |\sin x|\,dx$$

Split integral up at points where f changes sign.

$$= \underbrace{\int_0^{\pi} \sin x\,dx}_{\substack{|\sin x| = \sin x \\ \text{for } x\in[0,\pi]}} + \int_{\pi}^{2\pi} (-\sin x)\,dx.$$

$|\sin x| = -\sin x$ for $x \in [\pi, 2\pi]$

$$= 4$$

Next time:

Integration techniques - the substitution rule
(backwards chain rule)