Lecture 27

Today's topics:

- Properties of local extrema
- Shapes of graphs
- Sketching using derivatives

Read Ch. 5.6

Ex 5.6.1 - 5.6.7, 5.6.17 - 5.6.23, 5.6.35 - 5.6.41 } graph shapes from f', f''.

5.6.56, 57, 60, 65, 77 } sketching practice

Eoh 24

Tutorial: sketching practice

Properties of local extrema

→ Does a critical point c correspond to a local extrema? ($f'(c)=0$ OR $f'(c)$ DNE)

→ Find out using f'.

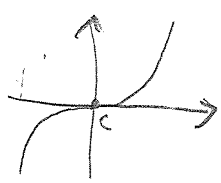
Note: c must belong to domain of f to be called a critical point

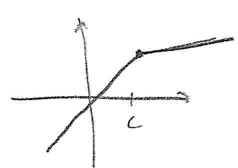
First Derivative Test

Evaluate sign of f' to the left and to the right of c.

Cases:

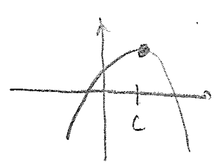
a) f' same sign on left & right

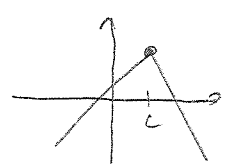




neither max nor min.

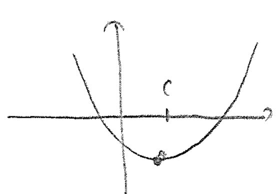
b) $f' > 0$ on left, $f' < 0$ on right

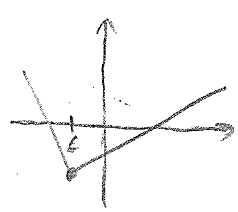




local max.

c) $f' < 0$ on left, $f' > 0$ on right





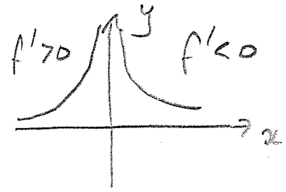
local min

Caution:

$f(c)$ must exist for it to be called an extrema.

Eg/ $f(x) = \frac{1}{x^2}$.

$f'(x) = -\frac{2}{x^3}$



f' DNE at $x=0$ but $x=0$ NOT a critical point as $0 \notin D_f$.

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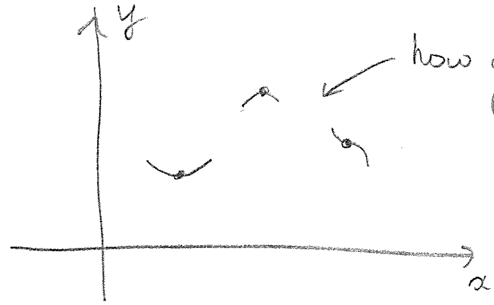
So far:

→ Algorithm for determining global extrema (using f')

→ Algorithm for shape of $f(x)$ at local extrema. (using f')

→ What else can derivatives tell us about the shape of $f(x)$?



how can we fill in the blanks?

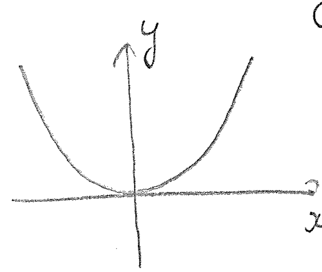
Increasing / Decreasing (formalising the intuitive)

→ Defn: $f(x)$ is strictly increasing on interval I if $f'(x) > 0$ for all $x \in I$.

→ Swap ($>$) with ($<$) for decreasing defn.

Eg. $f(x) = x^2$

$$f'(x) = 2x \begin{cases} >0, & x>0 \\ <0, & x<0 \end{cases}$$



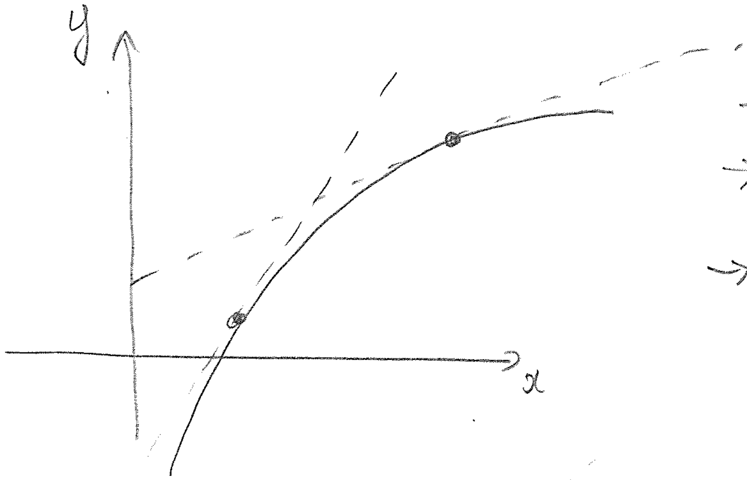
increasing on $(0, \infty)$

decreasing on $(-\infty, 0)$

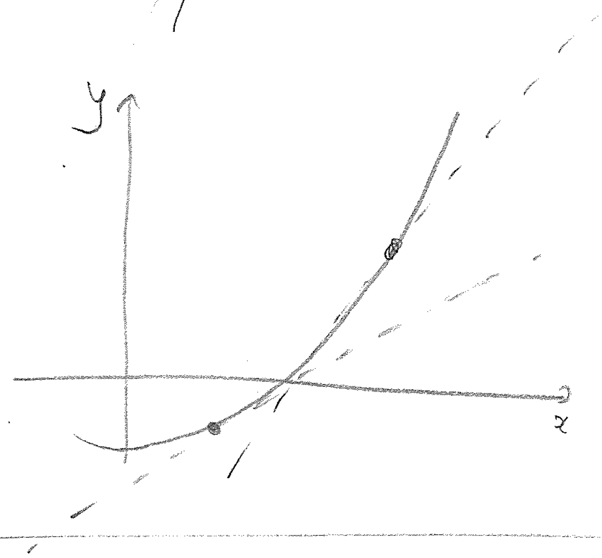
Concave up / Concave down

$f'(x)$: slope of tangent line to $f(x)$

$f''(x)$: change in slope.

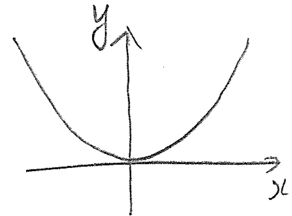


→ slope is decreasing
→ $f'' < 0$
→ concave down.



→ slope is increasing
→ $f'' > 0$
→ concave up.

Eg/. $f(x) = x^2$



$f'(x) = 2x$

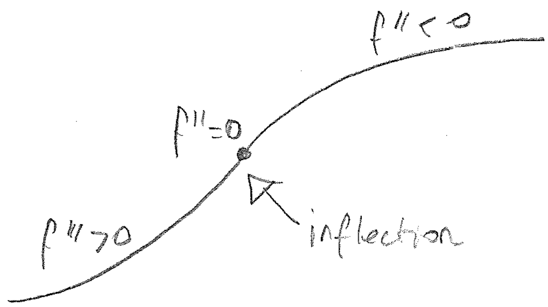
$f''(x) = 2 > 0$

$\Rightarrow$ f is concave up for $x \in \mathbb{R}$.

Q. What is f'' for a linear function?

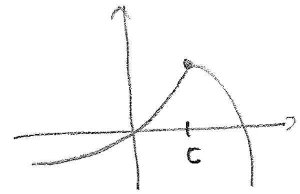
A. $y = mx + c$
$y' = m$
$y'' = 0$
(no change in slope)

Inflection Point



Defn: A point where a function switches concavities is an inflection point

either $f''(x) = 0$ OR $f''(x)$ DNE.



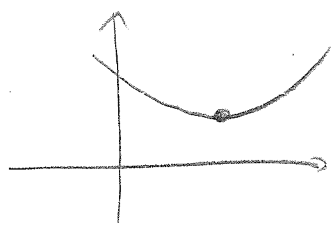
inflection with $f''(c)$ DNE.

The second-derivative test (uses f'')

→ Another test for properties of local extrema.

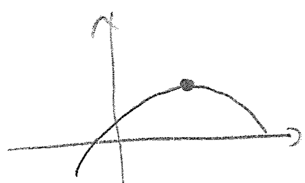
→ Use when f'' easy to compute.

a) If $f'(c) = 0$ and $f''(c) > 0$ then local min.



(concave up at local min)

b) If $f'(c) = 0$ and $f''(c) < 0$ then local max.



(concave down at local max)

Putting it all together for curve sketching

Sketch $f(x)$: things to consider

- domain
- range
- symmetry
- intercepts

} find from f

- asymptotes } find using limits

- critical points
- intervals of increase / decrease
- local max / min

} find using f'

- points of inflection
- intervals of concavity

} find using f''.

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* **Example (first derivative test)**

Q/ Find and characterize all local extrema of

$$f(x) = x^4 - 2x^2.$$

A/

$$f'(x) = 4x^3 - 4x$$
$$= 4x(x^2-1)$$
$$= 4x(x+1)(x-1)$$

$f' = 0$ at $x = 0, \pm 1$.

Investigate sign of f'