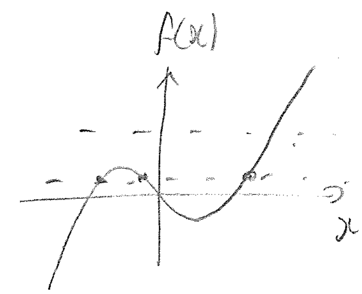
Lecture 10

Today's topics:
- trigonometry (part III)
  - inverse functions

Ch. 2.6
Examples 2.25 – 2.28
Exercises 2.6.1 – 2.6.3
2.8.10 – 2.8.13.

Functions without inverses



Classic problem: $f(x) = x^3 - x$.

Find x such that $f(x) = 0$.

What is $f^{-1}(0)$? f^{-1} doesn't exist!

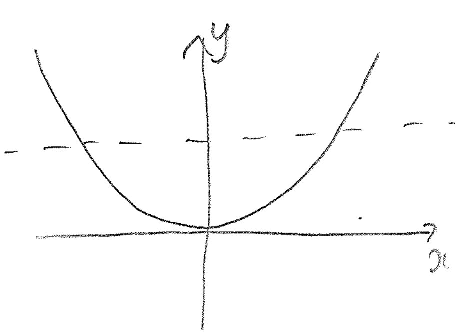
Solve noting $f(x) = x(x^2 - 1) = 0$

$\Rightarrow x = 0, \pm 1$.

Restricting the domain

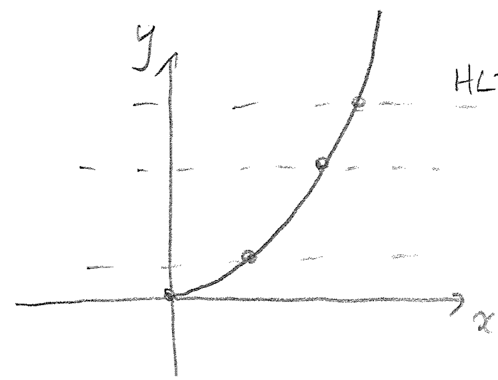
Consider $f(x) = x^2$

HLT fails



$\mathbb{D} = \mathbb{R}$

BUT



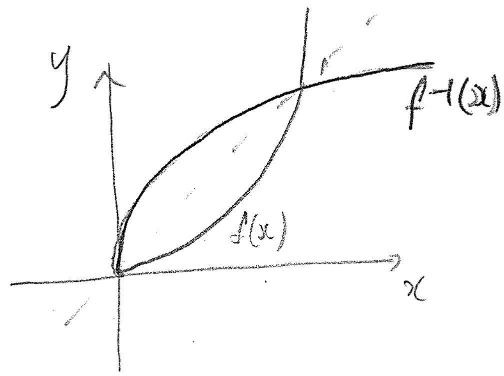
HLT pass.

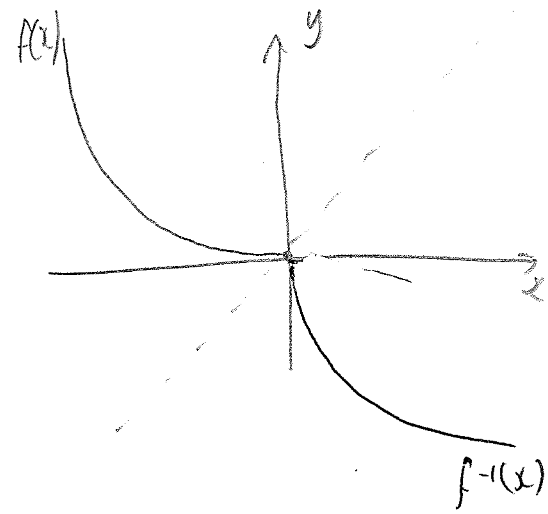
$\mathbb{D} = [0, \infty)$.

- Can restrict $\mathbb{D}$ to make $f(x)$ one-to-one
- Then f^{-1} exists!

Eg $f(x) = x^2$
$\mathbb{D} = [0, \infty) \Rightarrow f^{-1}(x) = \sqrt{x}$

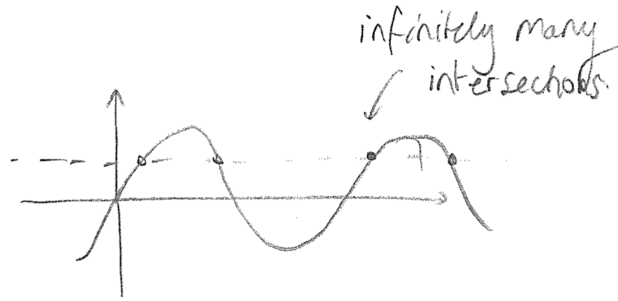
$f(x) = x^2$
$\mathbb{D} = (-\infty, 0] \Rightarrow f^{-1}(x) = -\sqrt{x}$





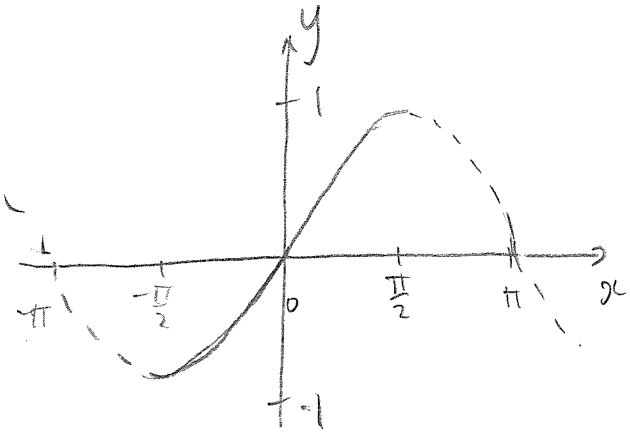
Trig Inverses

- Clearly not one-to-one
- $\sin(x) = \frac{\sqrt{3}}{2}$ has infinitely many solutions.



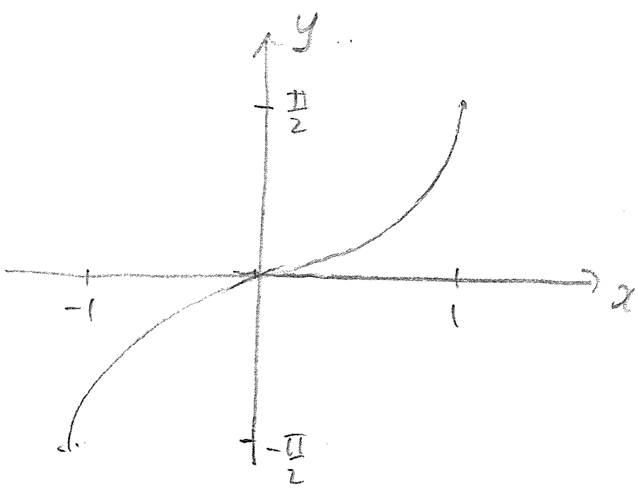
- Goal:- find subset of trig domain s.t graph is one-to-one.
  - establish "partial" inverses

The inverse of sine: $\arcsin(x) = \sin^{-1}(x)$



- Restrict to $\mathbb{D} = [-\frac{\pi}{2}, \frac{\pi}{2}]$.
- Makes sine one-to-one
- Same range, $[-1, 1]$.
- Each element in range has <u>one</u> element in domain

<u>Sketch:</u> $\mathbb{D} = [-1, 1]$, $\mathbb{E} = [-\frac{\pi}{2}, \frac{\pi}{2}]$



<u>Note:</u>

$$\sin\theta = \frac{opp}{hyp}$$

input: angle

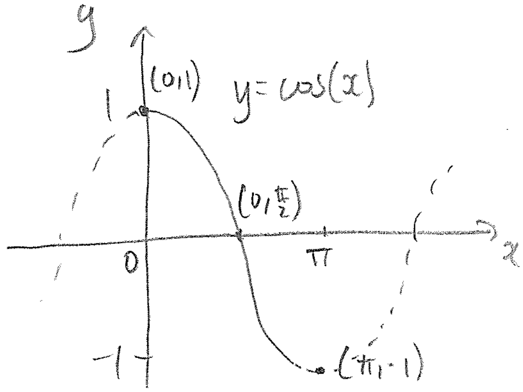
output: ratio of lengths

$$\arcsin(x) = \theta$$

input: ratio of lengths

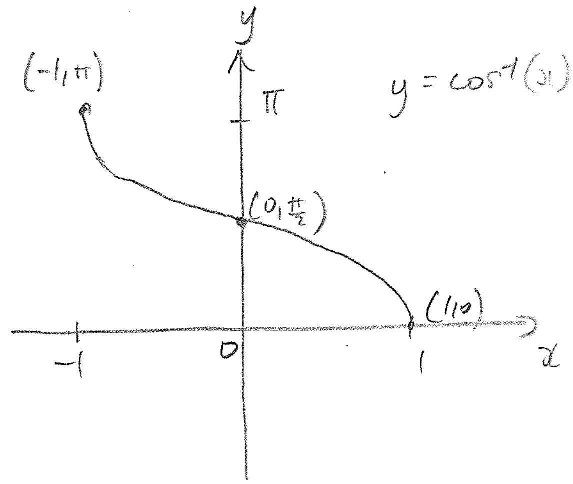
output: angle.

The inverse of cosine: $\cos^{-1}(x) = \arccos(x)$



Restrict to $\mathbb{D}=[0,\pi]$.

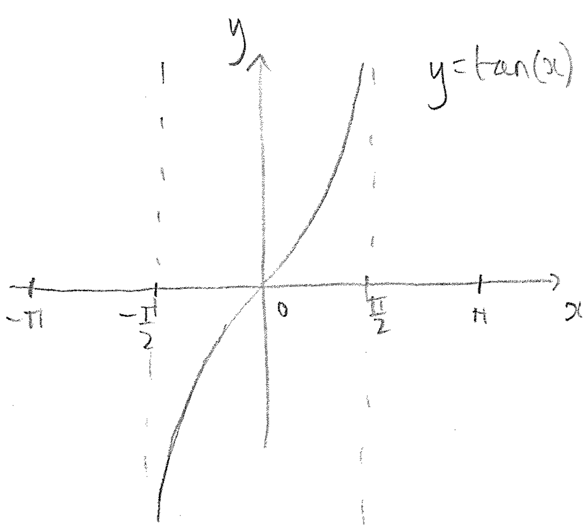
$\mathbb{E}=[-1,1]$.



$\mathbb{D}=[-1,1]$.

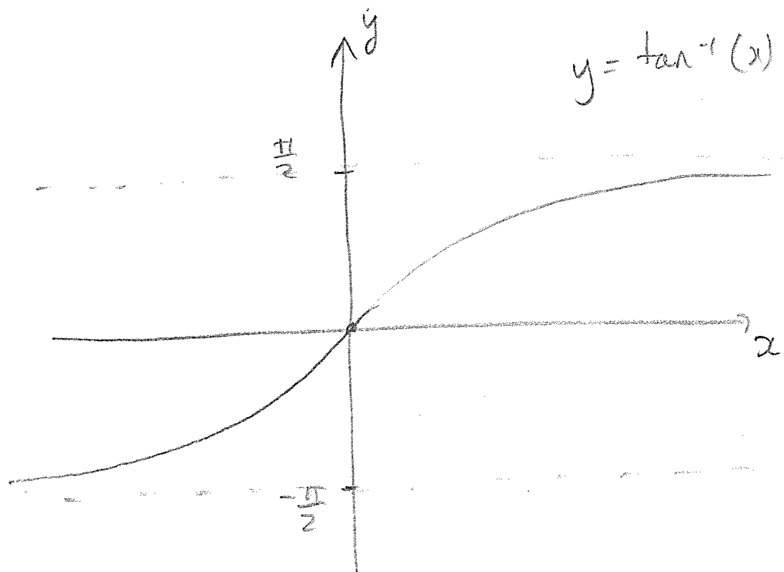
$\mathbb{E}=[0,\pi]$.

The inverse of tan: $\tan^{-1}(x) = \arctan(x)$



Restrict to $\mathbb{D}=(-\frac{\pi}{2},\frac{\pi}{2})$

$\mathbb{E}=(-\infty,\infty)$



$\mathbb{D}=(-\infty,\infty)$

$\mathbb{E}=(-\frac{\pi}{2},\frac{\pi}{2})$.

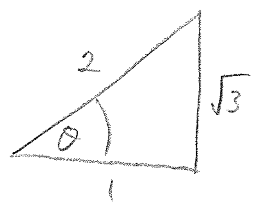
Example 1

Evaluate $\arcsin\left(\frac{\sqrt{3}}{2}\right)$.

Let $\theta = \arcsin\left(\frac{\sqrt{3}}{2}\right) \Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$

angle

and $\theta \in \underbrace{\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]}_{\text{range of arcsin}}$



special triangle

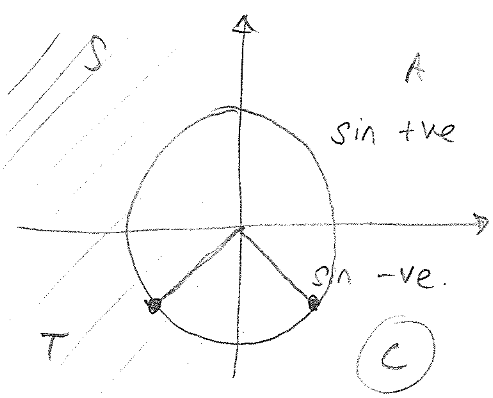
$\Rightarrow \theta = \frac{\pi}{3}$

always check to see if in range of arcsin!

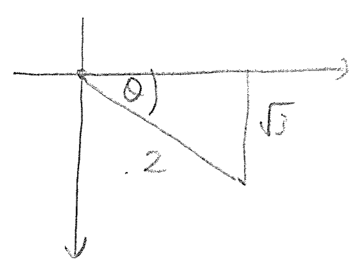
Example 2

Evaluate $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$.

Let $\theta = \arcsin\left(-\frac{\sqrt{3}}{2}\right) \Rightarrow \sin\theta = -\frac{\sqrt{3}}{2}$, $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$.



out of range



✓

$\theta = -\frac{\pi}{3}$

$\Rightarrow \arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$.

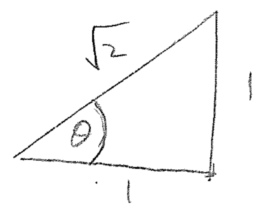
Example 3.

angle

↓

Evaluate $\sin\left(\arctan(1)\right)$

Let $\theta = \arctan(1) \Rightarrow \tan\theta = 1$ and $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$.



$\theta = \frac{\pi}{4}$ (in range)

$\sin\left(\arctan(1)\right) = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$