Announcements:- Office hours Tue 9.30-11.00
Thu 4.30-6.00

Lecture 4

Mobius bugs - posted on learn.
Textbook 2.6.6 - sol posted - Learn - Sec 003

Today's Topics:

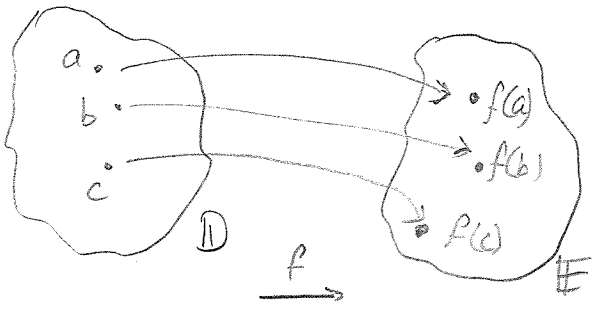
Inverse functions
- when they exist
- how to find them
- domains and ranges
- graphing

Extra:
Read Ch 2.4
Example 2.16, 2.17
Exercise 2.4.3

Reversing a function.

A function maps an input to an output

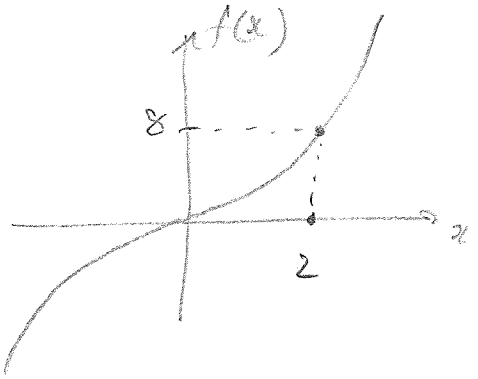
$x \longrightarrow \boxed{f} \longrightarrow f(x)$

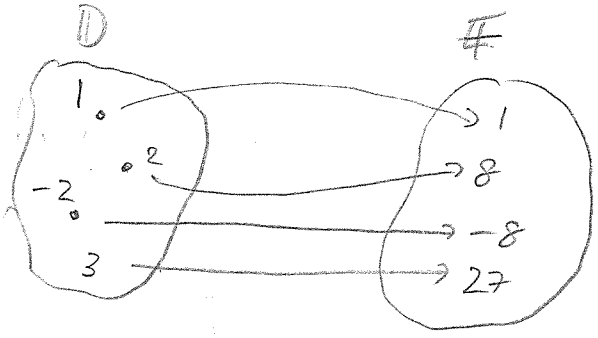


Is there a function that reverses the operation?

Sometimes there is

Eg. $f(x) = x^3$.



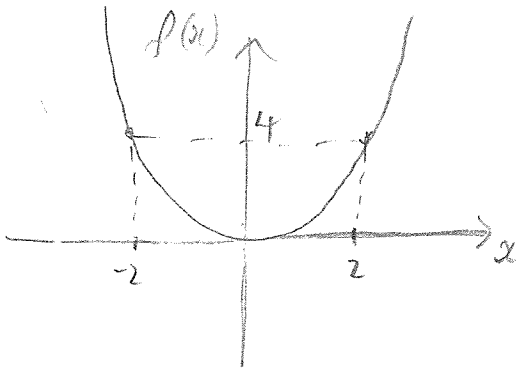


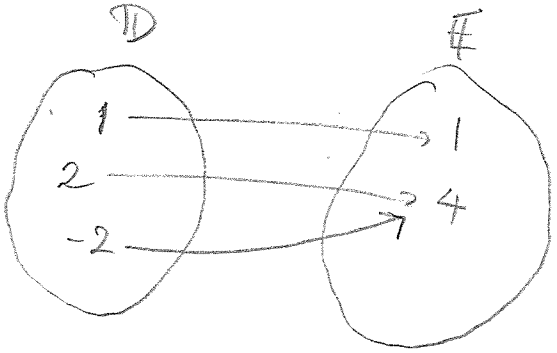
Every element in F can be mapped back to D uniquely.

$g(x) = \sqrt[3]{x}$. $f(2) = 8$; $g(8) = 2$.

Sometimes there isn't

Eg. $f(x) = x^2$.





$f(2) = 4$. $g(4) = 2, -2$?

no function can output 2 values for same input!

The inverse function. $(f^{-1}(x))$

$x \longrightarrow \boxed{f} \longrightarrow f(x)$.

$f(x) \longrightarrow \boxed{f^{-1}} \longrightarrow x$.

Note on notation.

$f^{-1}(x) \neq \frac{1}{f(x)}$. $\quad x^{-1} = \frac{1}{x}$ (reciprocal)

The Vital Property for f^{-1} to exist.

$f(x)$ must be a one-to-one function

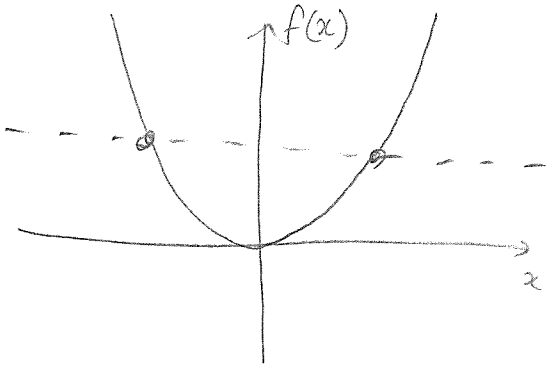
Def: $f(x)$ is one-to-one if

$$f(x_1) \neq f(x_2) \quad \text{whenever } x_1 \neq x_2$$

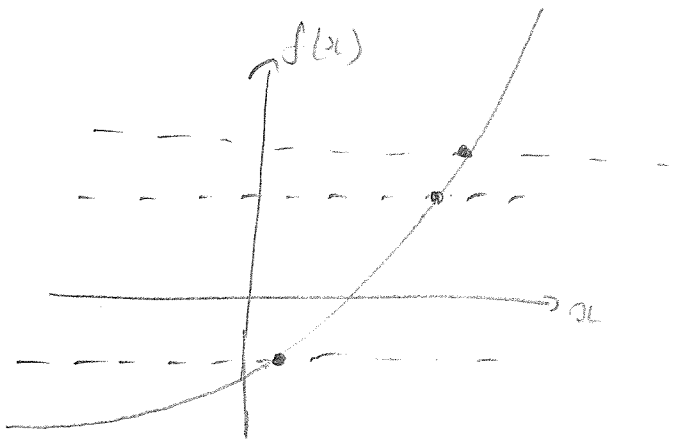
Eg/ $f(x) = x^2$ NOT one-to-one as $f(-2) = f(2)$.

Horizontal Line Test

$f(x)$ is one-to-one if no horizontal line intersects f more than once.



f not invertible



f invertible.

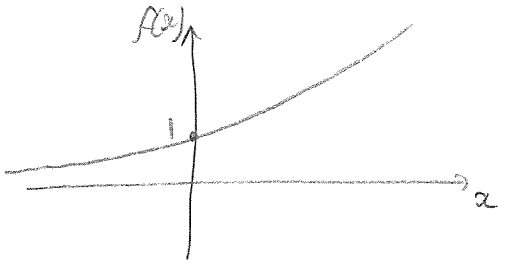
Domain and Range

Let $f : \mathbb{D} \to \mathbb{E}$ be one-to-one.

Then f^{-1} exists and has domain $\mathbb{E}$ and range $\mathbb{D}$. (switched).

$$y = f(x) \iff x = f^{-1}(y)$$

Eg. $f(x) = e^x$



$\mathbb{D}_f = \mathbb{R}$. $\mathbb{E}_f = (0, \infty)$

$\mathbb{D}_{f^{-1}} = (0, \infty)$ $\mathbb{E}_{f^{-1}} = \mathbb{R}$.

Finding an Inverse

Eg. $f(x) = \dfrac{4x-1}{2x+3}$

Step 1: Let $y = f(x)$

Step 2: Solve for x in terms of y.

Step 3: Rewrite as f^{-1}. - swap variables only if no physical meaning

$$y = \frac{4x-1}{2x+3}$$

$$(2x+3)y = 4x-1$$

$$2xy + 3y = 4x - 1$$

$$2xy - 4x = -3y - 1$$

$$x(2y-4) = -3y-1$$

$$x = \frac{3y+1}{4-2y}$$

$$f^{-1}(x) = \frac{3x+1}{4-2x}$$

Physical meaning?

Suppose height of a tree (h) at time t is given by

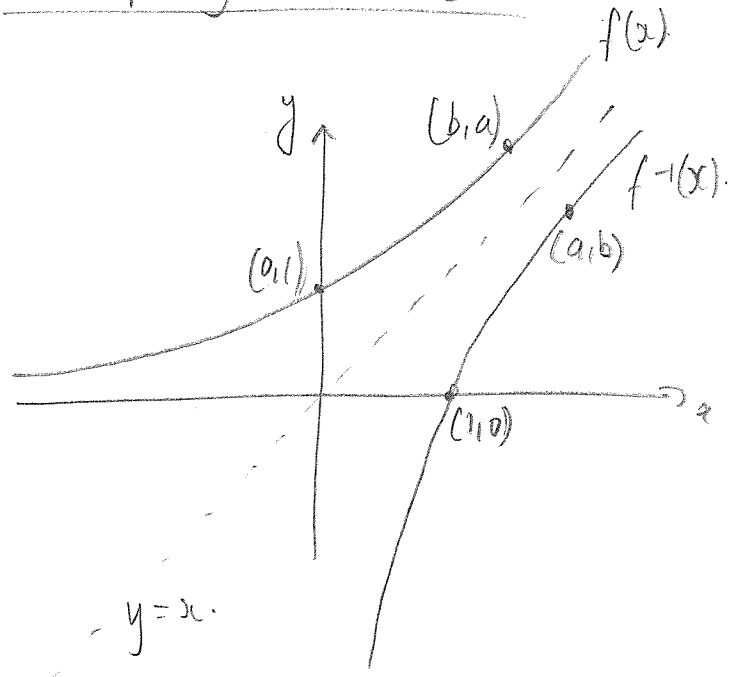
$$h(t) = \frac{7t}{1+t}, \quad t \geq 0.$$

Find time as a function of height

$$t(h) = \frac{h}{7-h}$$

→ don't swap variables!

Graphing Inverses



$y = f(x)$

$x = f^{-1}(y)$

graph of f^{-1} is a reflection of graph of f in the line $y = x$.