Early warning signals for bifurcations

QLSC 600
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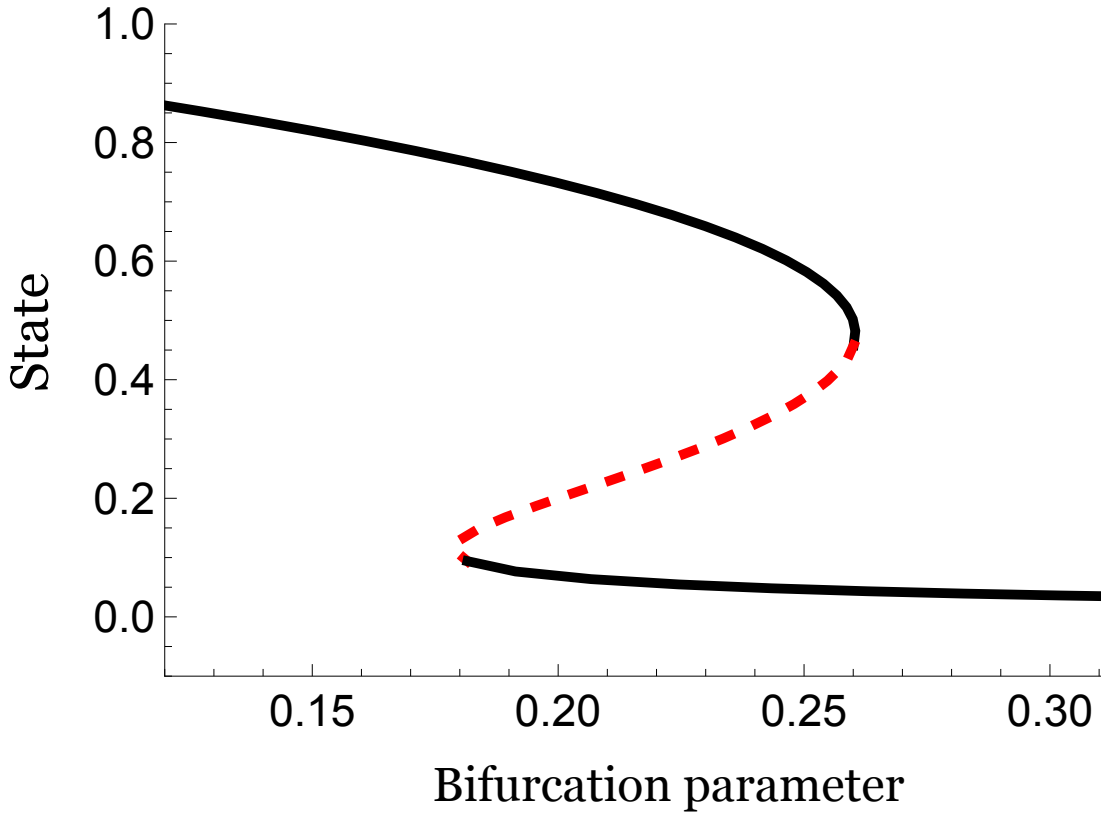
Instructor: Thomas Bury

Postdoctoral researcher
Department of Physiology
McGill University

Bifurcation (OED): The division of something into two branches or parts.

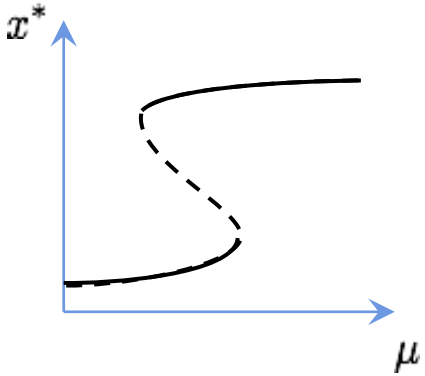
Bifurcation (dynamical systems): A point in parameter space where the qualitative dynamics of a system abruptly changes



What real-world systems can undergo an abrupt change in dynamics as a threshold is crossed?

Big data and bifurcations

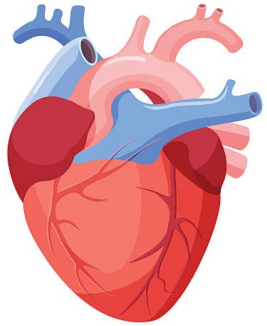
1.)

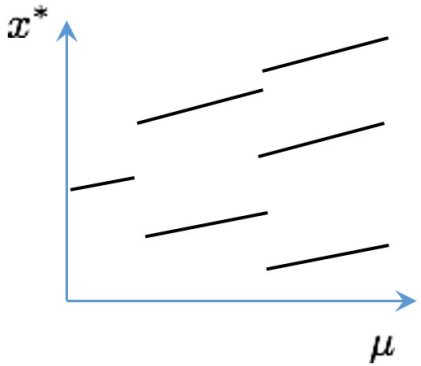


1 measurement / minute
1 year = **O(10^7)** data points

North Temperate Lakes
US Long-Term Ecological Research Network

2.)

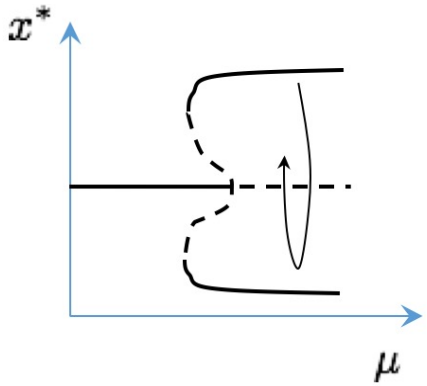


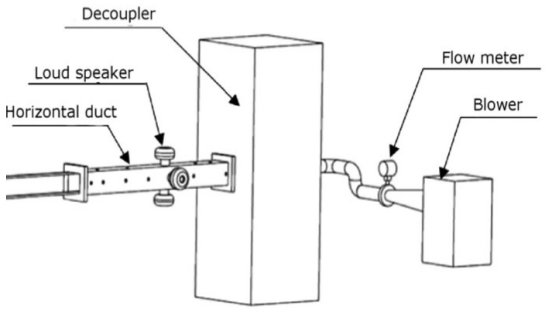


Sampling rate 250Hz
2 weeks = **O(10^8)** data points

icentia™

3.)





Sampling rate 10kHz
20 minutes = **O(10^7)** data points

R. I Sujith, IIT Madras

1. Can we learn the mechanisms that give rise to these bifurcations? (Why?)
2. Can we predict an upcoming bifurcation? (When?)
3. Can we predict the dynamics of an upcoming bifurcation? (What?)

Vol 461|3 September 2009|doi:10.1038/nature08227 nature

REVIEWS

Early-warning signals for critical transitions

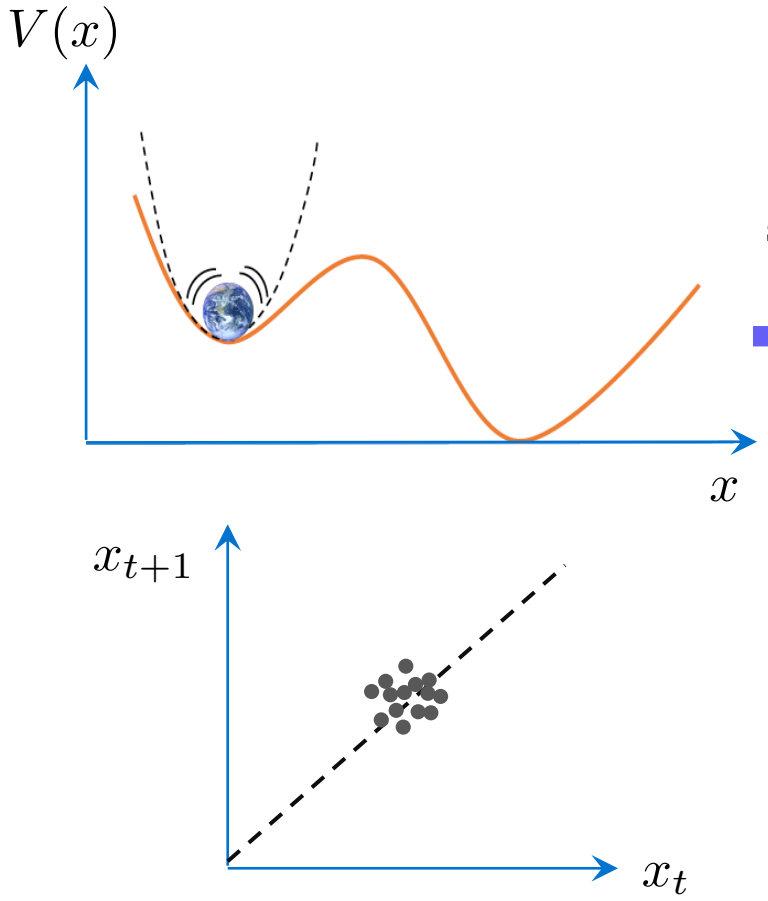
Marten Scheffer[1], Jordi Bascompte[2], William A. Brock[3], Victor Brovkin[5], Stephen R. Carpenter[4], Vasilis Dakos[1], Hermann Held[6], Egbert H. van Nes[1], Max Rietkerk[7] & George Sugihara[8]

Complex dynamical systems, ranging from ecosystems to financial markets and the climate, can have tipping points at which a sudden shift to a contrasting dynamical regime may occur. Although predicting such critical points before they are reached is extremely difficult, work in different scientific fields is now suggesting the existence of generic early-warning signals that may indicate for a wide class of systems if a critical threshold is approaching.

Early warning signals – intuitively

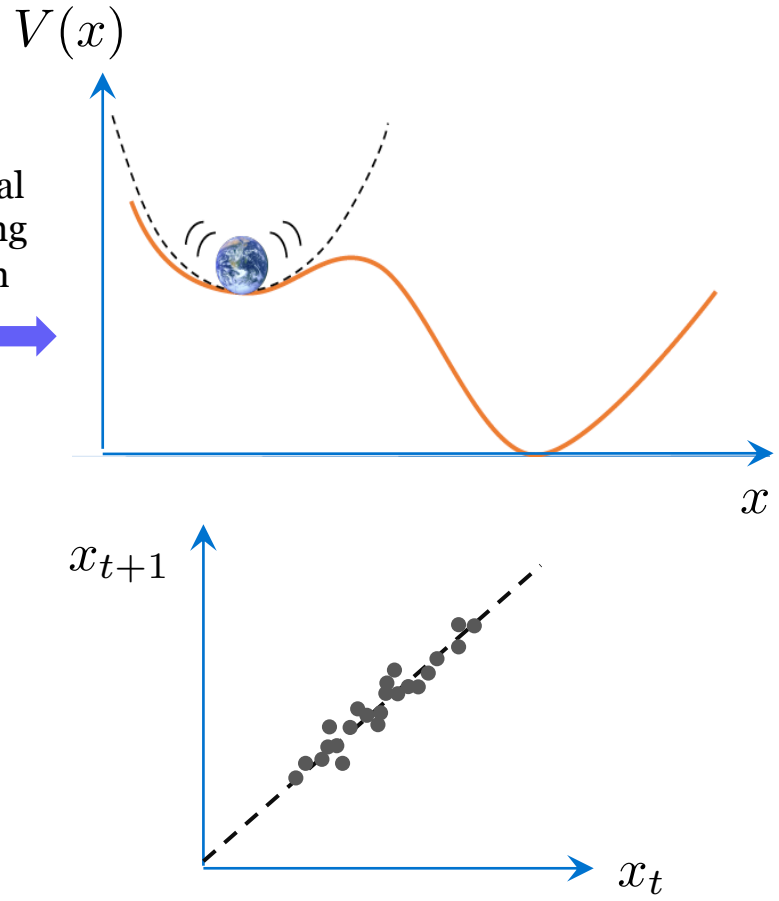
- Statistical metrics that warn of an approaching bifurcation

Far from bifurcation



Critical slowing down

Close to bifurcation



- Increase in recovery time from local perturbations

In **stochastic** systems:

- Increase in variance
- Increase in autocorrelation

Scheffer et al. **Early-warning signals for critical transitions** (Nature, 2009)

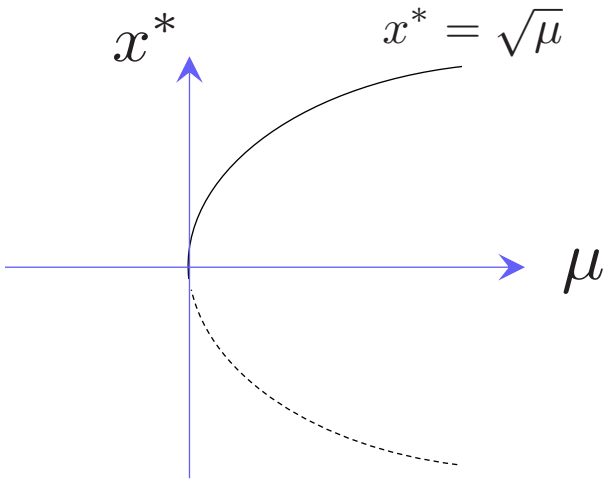
René Thom. **Structural stability, catastrophe theory, and applied mathematics** (SIAM review, 1977)

Early warning signals – mathematically

A simple system for illustration

$$\dot{x} = \mu - x^2 + \sigma\epsilon(t)$$

Normal form of the Fold bifurcation



Residual dynamics

$$y = x - x^*$$

satisfy

$$\dot{y} = -2\sqrt{\mu}y + \sigma\epsilon(t) + O(y^2)$$

Dropping H.O.T

$$\dot{y} = \lambda y + \sigma\epsilon(t)$$

an **Ornstein-Uhlenbeck Process**

The 'dominant eigenvalue'

$$\lambda = -2\sqrt{\mu}$$

Derive statistical metrics[1]

$$\mathrm{Var}(y) = \frac{\sigma^2}{2|\lambda|}$$

$$\rho(\tau) = e^{-|\lambda|\tau}$$

As the bifurcation is approached

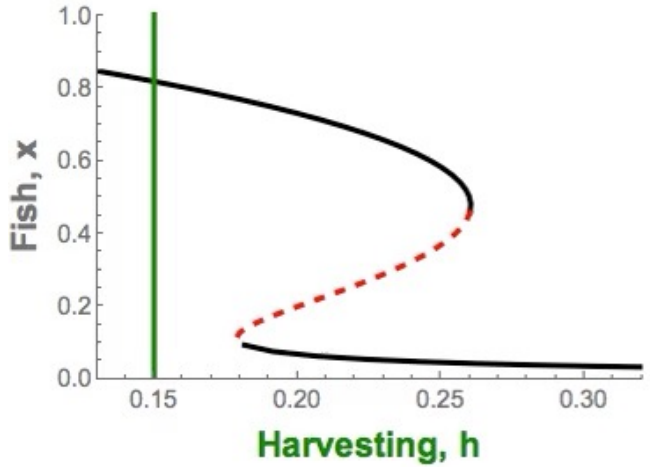
$(|\lambda| \to 0)$

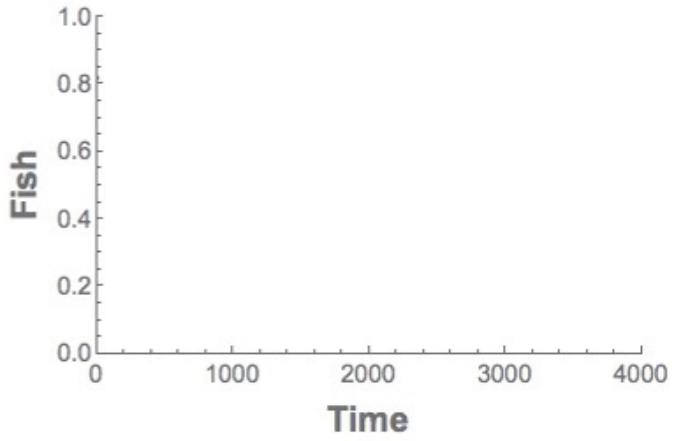
$$\mathrm{Var} \to \infty$$

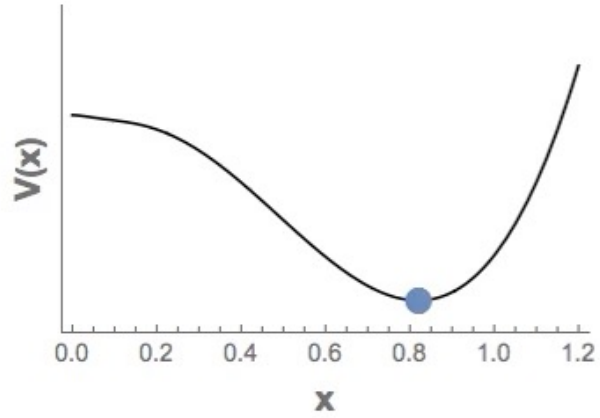
$$\rho \to 1$$

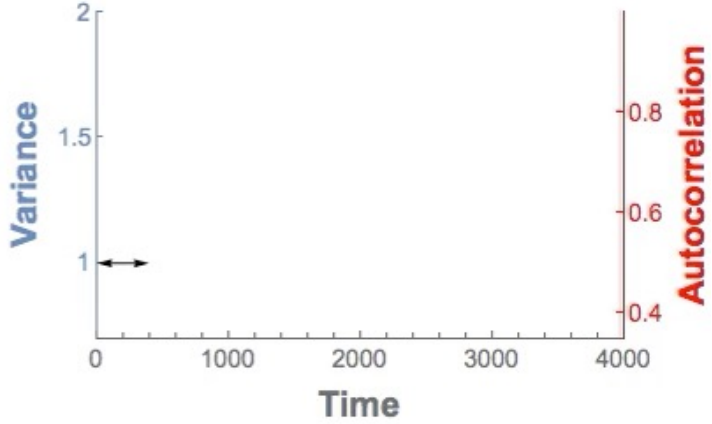
[1]Gardiner, **Handbook of stochastic methods,** Springer Berlin (1985)

Early warning signals – in action









May's harvesting model

$$\frac{dx}{dt} = x(1-x) - \frac{hx^2}{s^2+x^2} + \sigma\xi(t)$$

Early warning signals

Residual dynamics

$$y_t = x_t - \text{trend}$$

Variance

$$\text{Var}[y_t] = \langle y_t^2 \rangle$$

Lag-1 autocorrelation

$$\rho[y_t] = \frac{\langle y_t y_{t+1} \rangle}{\langle y_t^2 \rangle}$$

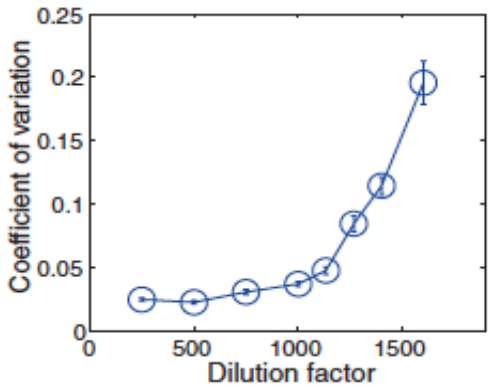
Potential function

$$V(x) = -\int^x f(\tilde{x})d\tilde{x}$$

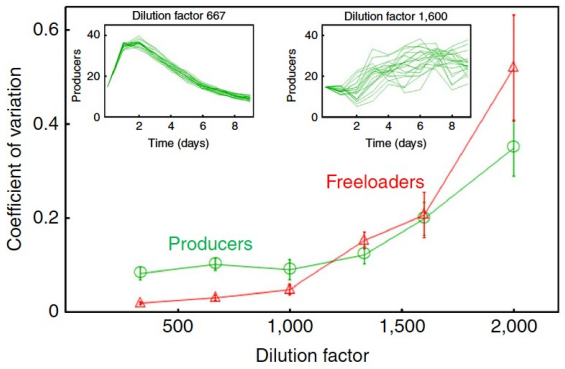
Robert May, **Threshold and breakpoints in ecosystems with a multiplicity of stable states.** Nature (1977)

Early warning signals – empirical observations

Microbial populations

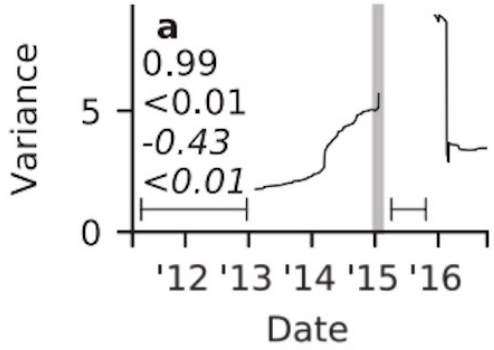


Dai et al. Science (2012)



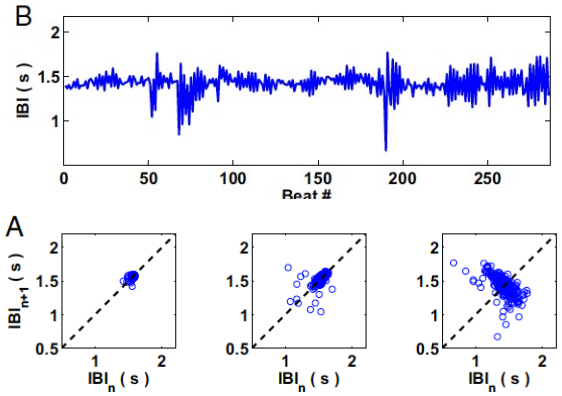
Chen et al. Nat. Comms (2012)

Tweets



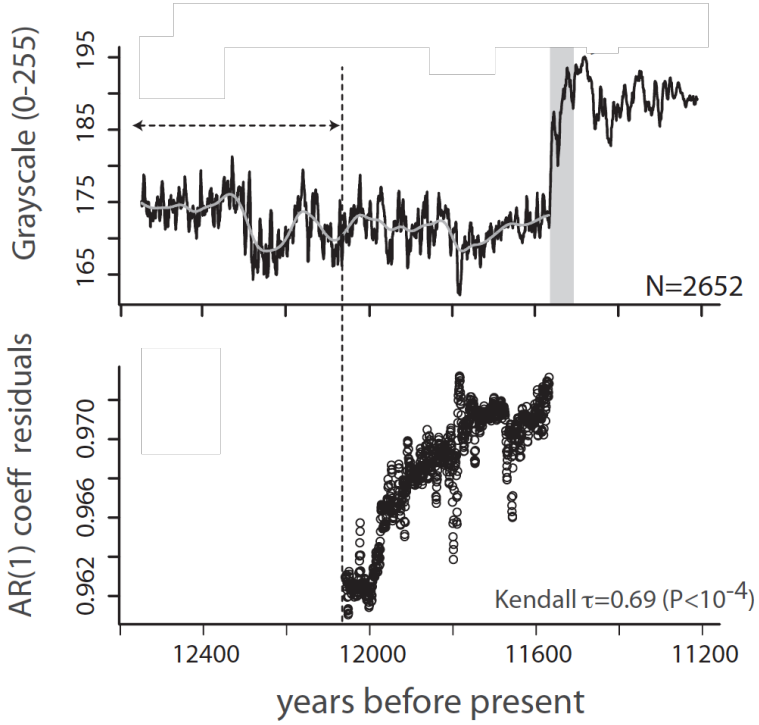
Pananos et al. PNAS (2017)

Cardiology



Quail et al. PNAS (2015)

Paleoclimate records



Dakos et al. PNAS (2008)

When might this type of early warning signal fail ?

PHILOSOPHICAL TRANSACTIONS B

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Resilience indicators: prospects and limitations for early warnings of regime shifts

Vasilis Dakos[1], Stephen R. Carpenter[2], Egbert H. van Nes[3] and Marten Scheffer[3]

Review

Limitations

- Enough data (>50 data points prior to transition)
- Bifurcation parameter needs to vary sufficiently slowly
- Abrupt change needs to be due to a local bifurcation

Procedure for computing early warning signals

- Detrend the data if nonstationary (e.g. moving average)
- Get residual dynamics
- Use a rolling window to compute statistical metrics (early warning signals)
- Determine if there is an increasing trend (using e.g. Kendall tau coefficient)

Have a go in the Jupyter notebook provided!

Class exercise